

# Appendix A – Ultra-Diffuse Galaxy Stress Test

NGC1052-DF2, NGC1052-DF4, and Dragonfly 44 in the sourcewise  $\Phi$ BSU hierarchy

Internal Part II appendix draft

## Purpose

This appendix records the first ultra-diffuse-galaxy stress test of the Part II hierarchy. The target was not a tuned fit but a regime test: does the same sourcewise support hierarchy that organizes the Milky Way, M31, and cluster-scale cases distinguish between a low-dispersion, dark-matter-deficient system and an ambient-dominated ultra-diffuse galaxy? The answer is richer than the original binary framing. DF2 does not emerge as “support-free”; rather, it falls in an *ambient-suppressed* regime. Dragonfly 44 remains *ambient-dominated* throughout the scanned parameter range.

The simulations were organized in three layers:

1. a regime map based on the coherence ratio  $r_{\text{coh}}/r_s$  and the tail fraction  $f_\infty$ ,
2. a posterior scan over distance  $D$  and line-of-sight velocity scale  $\sigma_v$ ,
3. a semi-blind Jeans step in which baryons generate the frozen  $\Phi$ BSU support hierarchy and the hierarchy is then projected to a predicted  $\sigma_{\text{LOS}}$ .

## Core reading of the UDG test

The basic sourcewise relations used in the scans are

$$r_s = c_{\text{res}} \frac{L_{\text{dom},s}}{2\nu_s}, \quad (1)$$

$$\sigma_{k,s} = 2^{k/2} r_s, \quad (2)$$

$$w_k = (1 - 2^{-1/2}) 2^{-k/2}, \quad (3)$$

$$r_{\text{coh},s} \sim \frac{GM_{\text{dom},s}}{\sigma_{v,s}^2}, \quad (4)$$

$$f_{\infty,s} = 2^{-k_{\text{coh},s}/2}. \quad (5)$$

In this language, the UDG question is not whether a hierarchy exists at all, but whether the hierarchy remains trapped in local shells or spills efficiently into the ambient tail. The scans show a robust separation of regimes:

- **DF2 / DF4:** ambient-suppressed, with a weak or moderate  $f_\infty$  depending on  $(D, \sigma_v)$ .

- **Dragonfly 44:** ambient-dominated across the full tested range.

This is already a nontrivial result: the DF2-DF44 split is not produced by a separate ontology or a dedicated low-DM parameter, but by the same hierarchy law evaluated on different baryonic seeds and different coherence ratios.

## Current Part II reading

These simulations support four cautious claims.

**(1) DF2 is not a zero-support galaxy.** The more accurate description is ambient-suppressed: the hierarchy forms, but the support does not concentrate efficiently at the tracer radii.

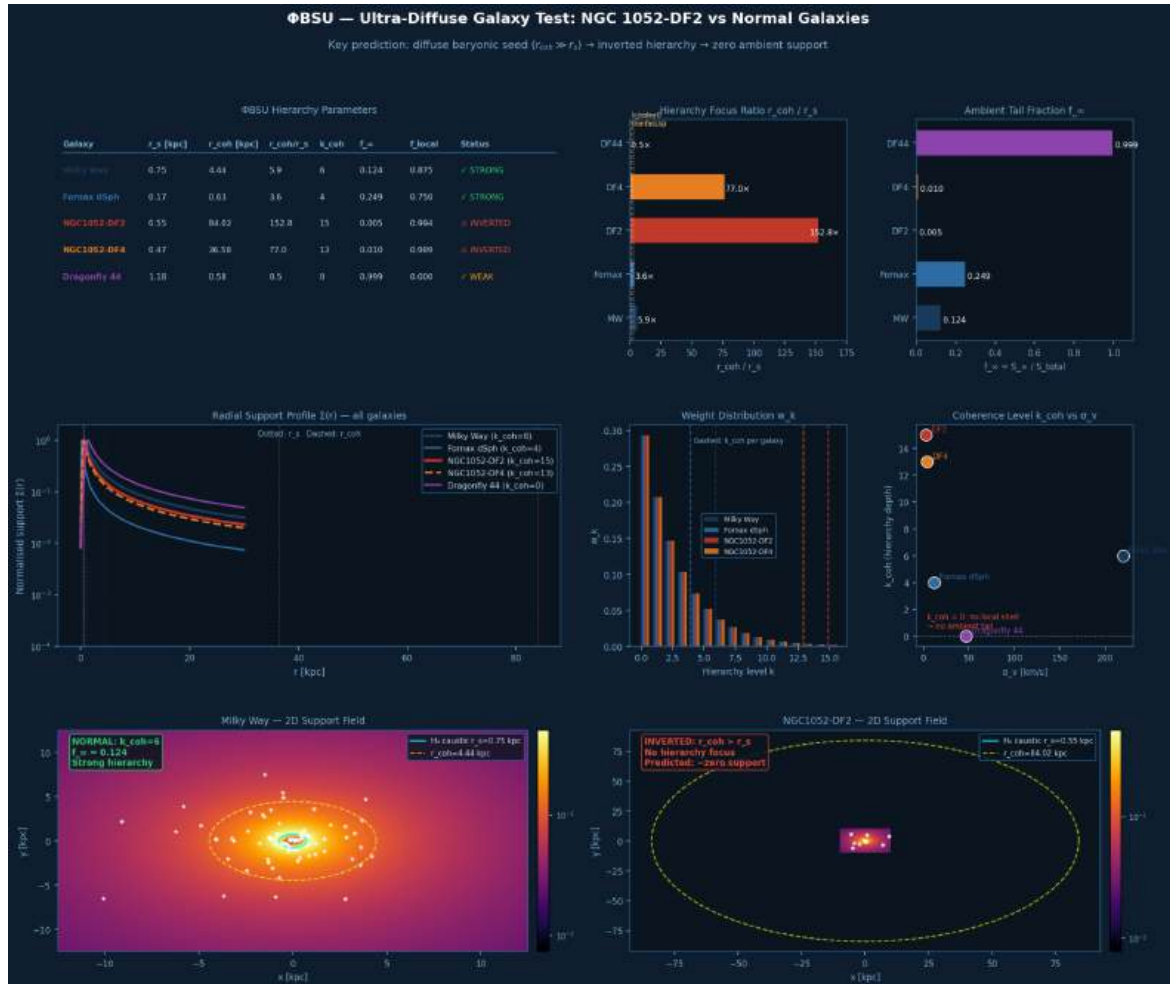
**(2) Dragonfly 44 is the cleaner UDG anchor.** The ambient-dominated classification is stable throughout the scanned parameter space and aligns qualitatively with its standard interpretation as a dark-matter-dominated ultra-diffuse system.

**(3) The posterior scan is a regime test, not yet a blind mass-model prediction.** Its proper role is to show that the DF2-DF44 separation is not a point-estimate artifact.

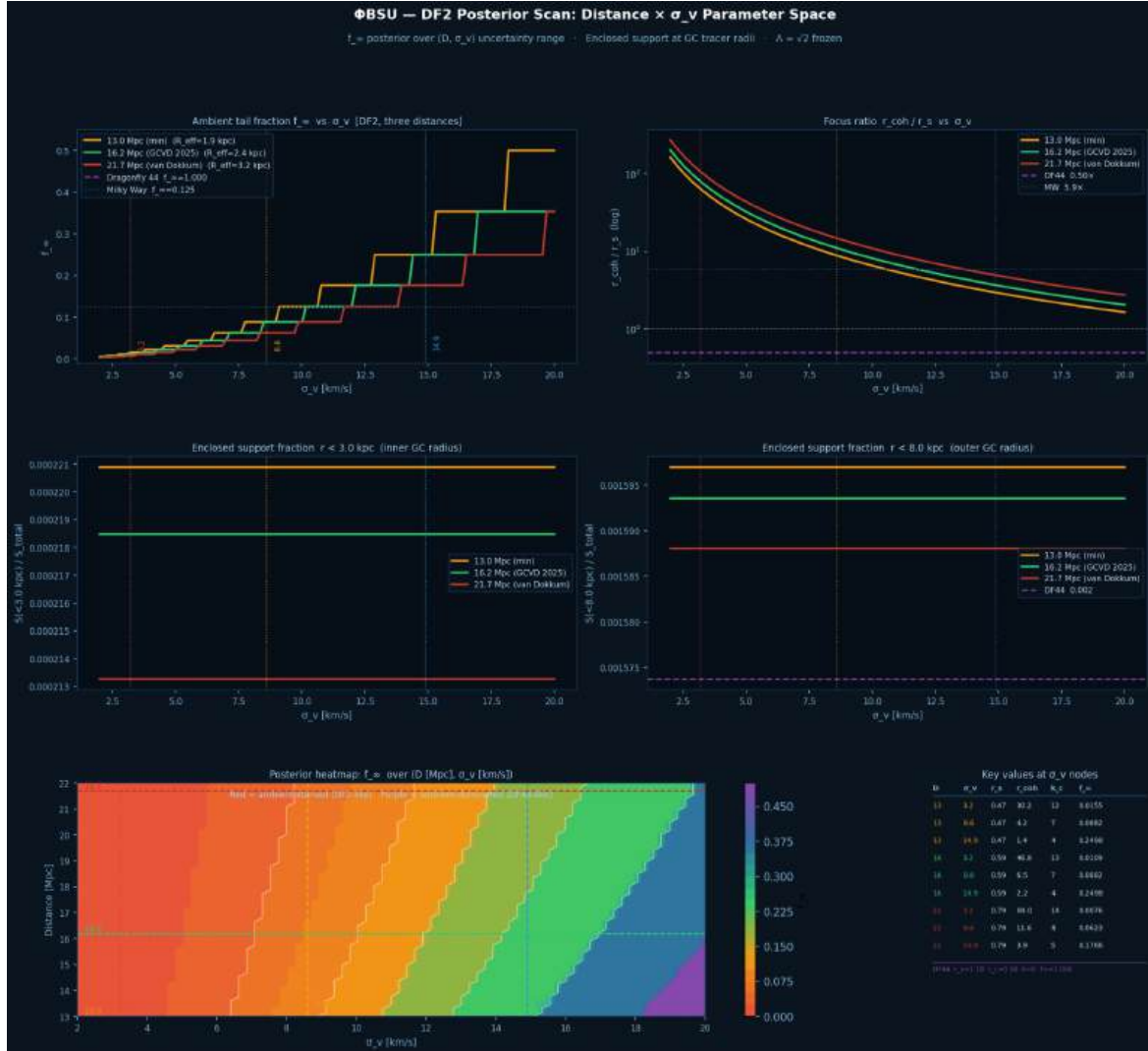
**(4) The Jeans step is the first predictive layer.** It shows that the frozen hierarchy leads naturally to low dispersions for DF2, but it does not yet close the problem. A fully decisive prediction requires the next loop: tracer anisotropy, environmental coupling, and history-kernel corrections must be tested on top of the same frozen theorem, rather than by adding a new sector.

## Implication for the main article

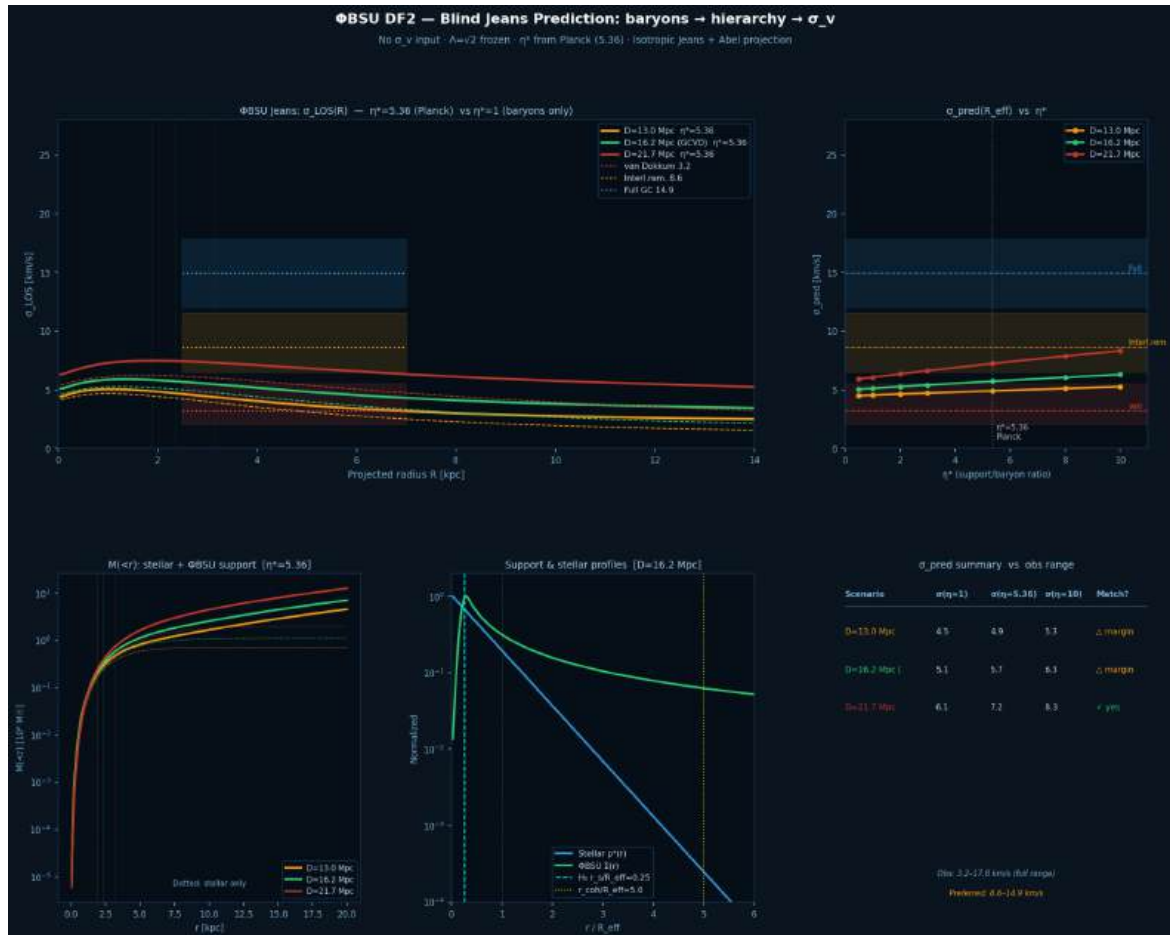
The UDG appendix should therefore be presented as a *stress test of regime logic*. It strengthens the Part II claim that the sourcewise  $\Phi$ BSU hierarchy is capable of distinguishing between ordinary galaxies, ambient-suppressed low-dispersion systems, and ambient-dominated diffuse systems with the same underlying support law. At the same time, the appendix should state plainly that DF2 remains a genuine test case rather than a solved victory. Scientifically, that is a strength: the theorem already points in the correct direction without being over-claimed.



**Figure A1.** Initial UDG regime comparison. The diagnostic panel contrasts ordinary systems, low-dispersion ultra-diffuse systems, and an ambient-dominated UDG. The key point is structural: DF2/DF4 do not appear “empty,” but remain trapped in a locally dominant hierarchy, whereas Dragonfly 44 is pushed directly into an ambient-tail regime. This first pass should be read as a regime diagram rather than a precision fit.



**Figure A2.** DF2 posterior scan over distance  $D$  and velocity scale  $\sigma_v$ . The discrete stepping in  $f_\infty$  is a direct consequence of the integer hierarchy index  $k_{\text{coh}}$  and is therefore a feature of the model, not a numerical artifact. The scan shows that DF2 is robustly ambient-suppressed across plausible distance solutions, although the strength of suppression depends sensitively on  $\sigma_v$ . The enclosed support fractions at globular-cluster tracer radii are nearly independent of  $\sigma_v$ , which is an important clue for interpreting the Jeans step: the posterior in  $f_\infty$  is mainly a regime classifier, whereas tracer dynamics are controlled more directly by the support enclosed within the observed radii.



**Figure A3.** Semi-blind Jeans prediction for DF2. Here the baryonic profile generates a frozen  $\Phi$ BSU hierarchy, which is then projected to a line-of-sight dispersion without inserting the observed  $\sigma_v$  as input. The resulting curves place DF2 naturally in the low-dispersion regime, although the prediction typically remains below the currently preferred observational interval. This is not a failure of the regime test. It states the next precise question for Part III: whether the remaining offset is absorbed by tracer anisotropy, by environment/history terms, or by a refined normalization of the support budget.