Appendix A - Ultra-Diffuse Galaxy Stress Test

NGC1052-DF2, NGC1052-DF4, and Dragonfly 44 in the sourcewise ΦBSU hierarchy

Internal Part II appendix draft

Purpose

This appendix records the first ultra-diffuse-galaxy (UDG) stress test of the Part II hierarchy. The target is not a tuned fit but a regime test: does the same sourcewise support hierarchy that organizes the Milky Way, M31, and cluster-scale cases distinguish between a low-dispersion, dark-matter-deficient system and an ambient-dominated ultra-diffuse galaxy? The answer is richer than the original binary framing. DF2 does not emerge as support-free; rather, it falls in an *ambient-suppressed* regime. Dragonfly 44 remains a *high-tail* control case across the scanned parameter space, even after revised kinematic inputs are folded in.

Core reading of the UDG test

The sourcewise relations used throughout the scans are

$$r_s = c_{\mathrm{res}} \frac{L_{\mathrm{dom},s}}{2\nu_s}, \tag{A1}$$

$$\sigma_{k,s} = 2^{k/2} r_s, \tag{A2}$$

$$w_k = \left(1 - 2^{-1/2}\right) 2^{-k/2}, \tag{A3}$$

$$r_{\mathrm{coh},s} \sim \frac{G M_{\mathrm{dom},s}}{\sigma_{v,s}^2}, \tag{A4}$$

$$f_{\infty,s} = 2^{-k_{\mathrm{coh},s}/2}. \tag{A5}$$

In this language, the UDG question is not whether a hierarchy exists at all, but whether the hierarchy remains trapped in local shells or spills efficiently into the ambient tail. The simulations support three cautious claims:

- **DF2 / DF4:** ambient-suppressed, with weak or moderate f_∞ depending on (D, σ_v).
- **Dragonfly 44:** a high-tail comparison case, ambient-dominated in the original benchmark and still clearly separated from DF2 after revised kinematics.
- **Jeans layer:** the frozen hierarchy places DF2 naturally in the low-dispersion regime without inserting the observed σ_v as an input, although the prediction still tends to fall below the currently preferred interval.

Current Part II reading. The UDG appendix should be presented as a *stress test of regime logic*. It strengthens the claim that the same sourcewise ΦBSU hierarchy distinguishes between ordinary galaxies, ambient-suppressed low-dispersion systems, and high-tail diffuse systems without introducing a separate low-DM ontology. At the same time, DF2 remains a genuine test case rather than a solved victory.

The van Dokkum acid test

The original van Dokkum 2016 DF44 parameters are retained as a deliberate acid test. They define the strongest plausible ambient-dominated corner of UDG parameter space: if the hierarchy only worked for that legacy point, the separation would be fragile. The correct question is therefore not whether DF44 stays at $f_\infty = 1$ under every update, but whether the DF2/DF44 regime split survives after the DF44 anchor is weakened.

That is precisely what the revised scan shows. With the updated spatially resolved estimate $\sigma_v = 33\,\mathrm{km\,s^{-1}}$ and the originally used stellar mass, DF44 moves from the pure ambient-dominated corner to the ambient-dominated / transitional boundary, with $k_{\mathrm{coh}} = 1$ and $f_\infty \approx 0.71$. If the same revision is accompanied by the lower stellar mass implied by the updated globular-cluster census, the ambient-dominated classification is recovered naturally, with $k_{\mathrm{coh}} = 0$ and $f_\infty \approx 1$. In all tested cases DF44 remains well above the DF2 posterior band.

The role of the van Dokkum benchmark is therefore methodological. It functions as an acid test of *regime stability*: the anchor is intentionally weakened, and the hierarchy is asked whether the DF2 / DF44 split survives. The present answer is yes. The label attached to DF44 softens from pure ambient domination to a boundary case in the most conservative revised scenario, but the regime separation itself is not erased.

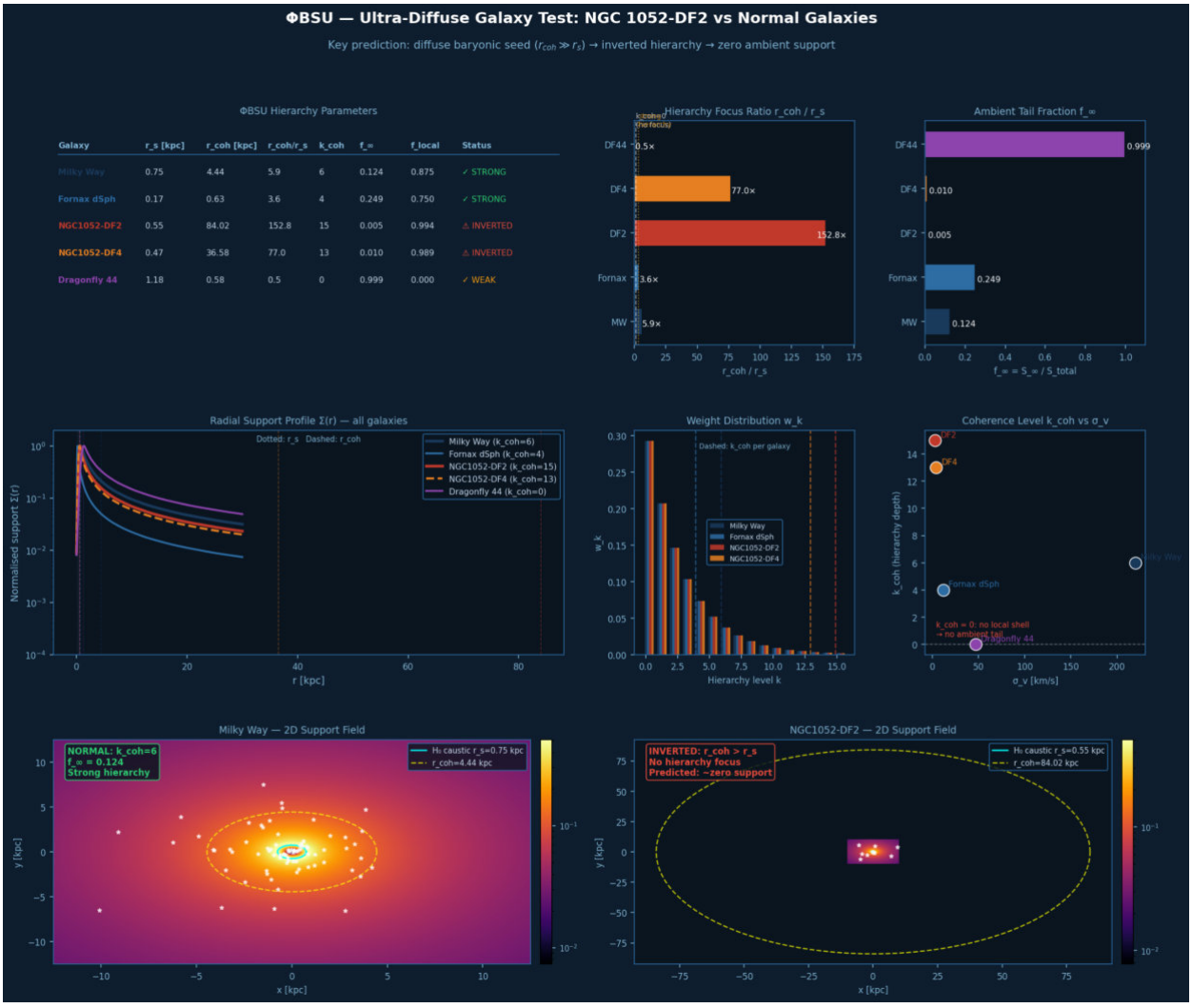

Figure A1: Initial UDG regime comparison. The diagnostic panel contrasts ordinary systems, low-dispersion ultra-diffuse systems, and an ambient-dominated UDG. The key point is structural: DF2 and DF4 do not appear support-free, but remain trapped in a locally dominant hierarchy, whereas Dragonfly 44 is driven toward the high-tail regime. This first pass should be read as a regime diagram rather than a precision fit. In the present appendix, the original van Dokkum DF44 point is treated as an upper-regime benchmark that is stress-tested in Figure A4 against revised kinematics.

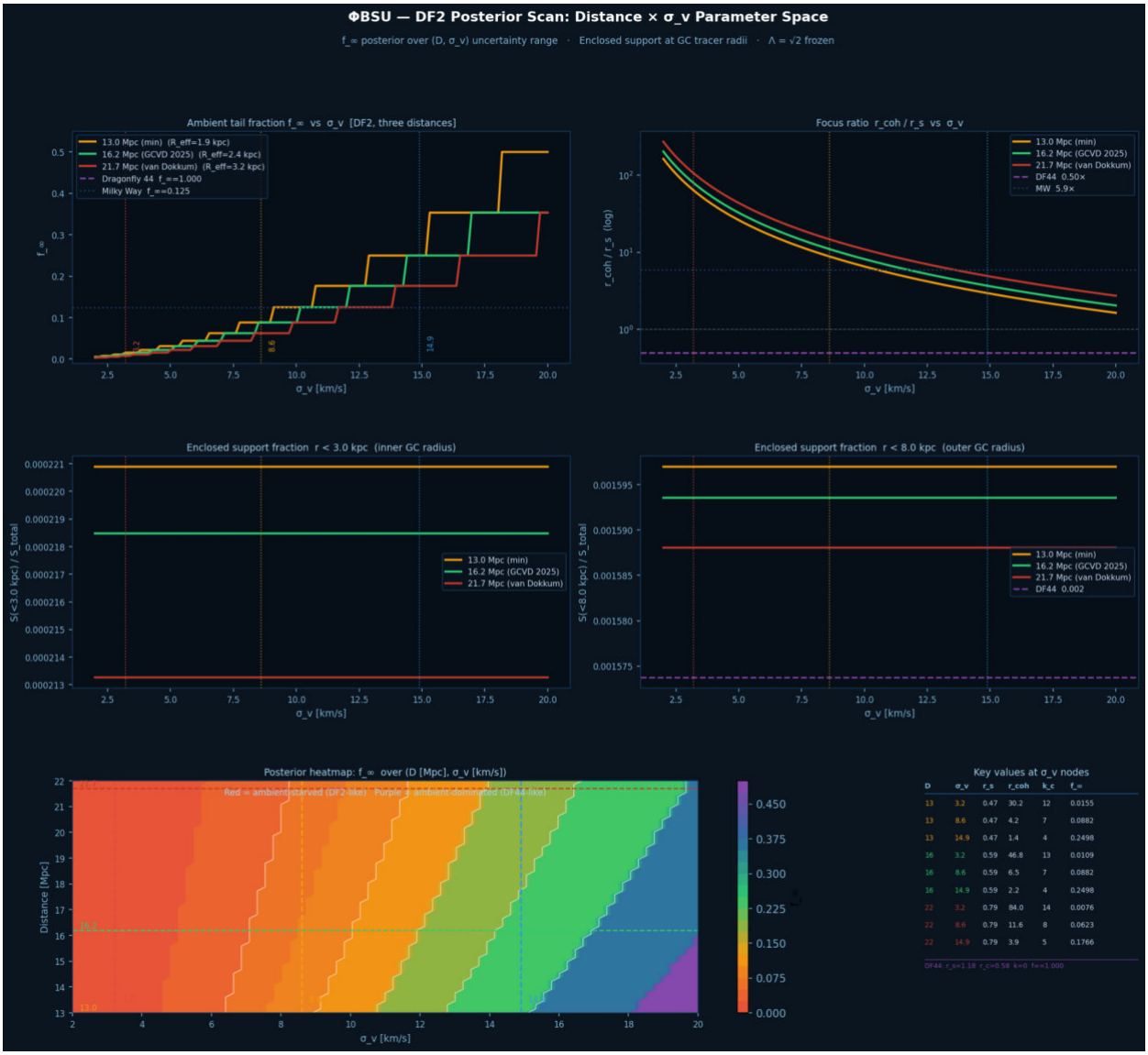

Figure A2: DF2 posterior scan over distance D and velocity scale σ_v. The discrete stepping in f_∞ is a direct consequence of the integer hierarchy index k_{coh} and is therefore a feature of the model rather than a numerical artifact. The scan shows that DF2 is robustly ambient-suppressed across plausible distance solutions, although the strength of suppression depends sensitively on σ_v. The enclosed support fractions at globular-cluster tracer radii are nearly independent of σ_v, which is an important clue for interpreting the Jeans step: the posterior in f_∞ is mainly a regime classifier, whereas tracer dynamics are controlled more directly by the support enclosed within the observed radii.

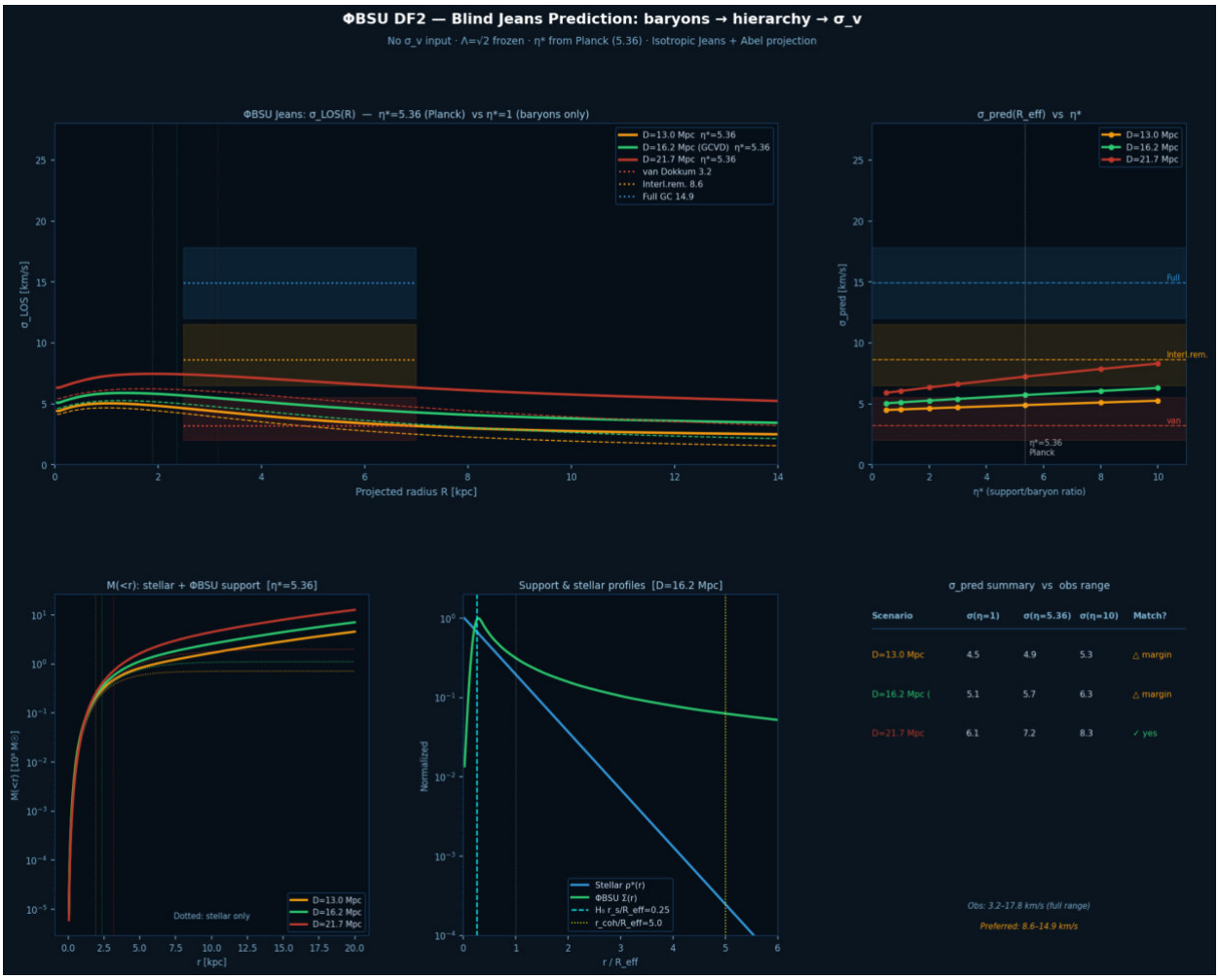

Figure A3: Semi-blind Jeans prediction for DF2. The baryonic profile generates a frozen ΦBSU hierarchy, which is then projected to a line-of-sight dispersion without inserting the observed σ_v as an input. The resulting curves place DF2 naturally in the low-dispersion regime, although the prediction typically remains below the currently preferred observational interval. This is not a failure of the regime test. It states the next precise question for Part III: whether the remaining offset is absorbed by tracer anisotropy, by environment/history terms, or by a refined normalization of the support budget.

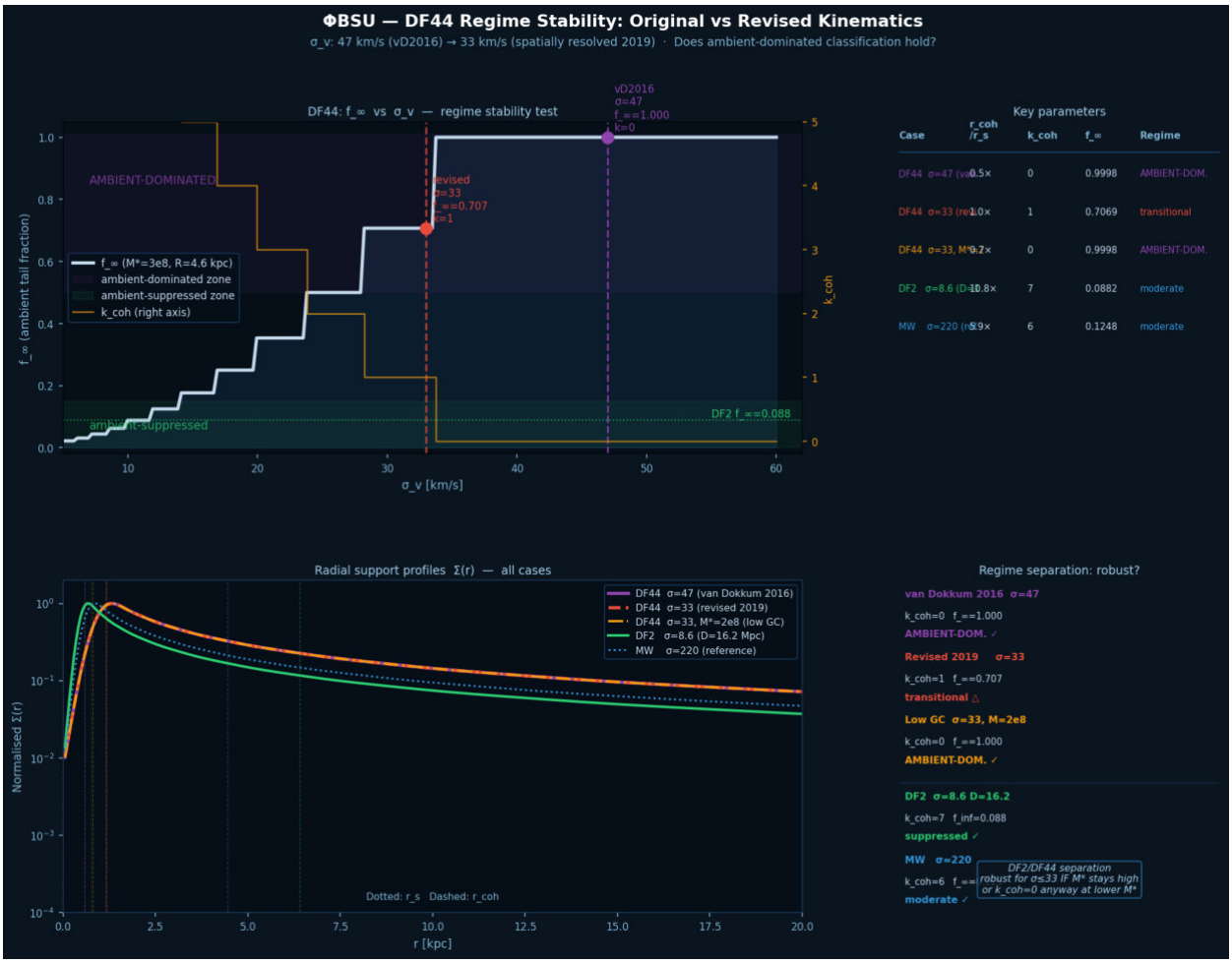

Figure A4: DF44 revised kinematics and regime stability. The original van Dokkum 2016 parameters act here as an acid test of the upper-regime anchor. With $\sigma_v = 47\,\mathrm{km\,s^{-1}}$, DF44 sits at the pure ambient-dominated corner, with $k_{\mathrm{coh}} = 0$ and $f_\infty \approx 1$. Using the revised $\sigma_v = 33\,\mathrm{km\,s^{-1}}$ estimate while retaining the higher stellar mass moves the system to the ambient-dominated / transitional boundary, with $k_{\mathrm{coh}} = 1$ and $f_\infty \approx 0.71$. If the same revision is combined with the lower stellar mass implied by the updated globular-cluster census, the ambient-dominated reading is recovered. The important result is not the exact label attached to DF44, but the stability of the DF2 / DF44 regime split under this deliberate weakening of the benchmark point.