Quark and Gluon Interior States from a 3^2 Holonomy Node

A Stand-Alone ΦBSU Construction of Fractional Charge, Hadron Closures, and Current-Tail Mass Ratios

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Abstract

This note isolates a quark–gluon research component from the broader ΦBSU programme and presents it as a stand-alone construction. The only background assumptions used are a global phase identity channel, a physical projection branch, and a 3^2 internal holonomy node. The key proposal is that the quark sector should not be modelled as three generations of closed point particles. Rather, quark-like states are colour-indexed partial holonomy terminals of a 3^2 node, while observable hadrons are invariant closures of those terminals; ordinary and exotic hadrons are therefore different invariant contractions rather than counterexamples to quark non-asymptoticity. The algebraic centre of the proposal is

$$\mathrm{End}(\mathbb{C}^3) \simeq \mathbb{C}I \oplus \mathfrak{sl}(3, \mathbb{C}),$$

whose compact operational chart gives a neutral running node plus eight gluon-like traceless directions. This yields a natural language for colour, gluon control, mesons, baryons, and exotic hadrons as invariant contractions.

For current-quark mass parameters we test a π-free C_3 trace-plane model. Branch orientations are encoded by cubic invariants

$$J_d = 1 - \frac{1}{2 \cdot 3^2} = \frac{17}{18}, \qquad J_u = 1 - \frac{2}{3^4} = \frac{79}{81},$$

while the gluon-node cycle opening is fixed from an up-branch edge condition and projected to the down branch by 3/8. With discrete scale-bridge rules

$$G_d = \frac{56}{27}, \qquad G_u = \frac{145}{2},$$

the resulting masses are close to the Particle Data Group consensus mass parameters for u, d, s, c, b, t. The quantum-state compatibility conditions and the scale-free mass ratios within each branch are derived inside the hypothesis. An inverted curvature check shows that the effective internal openings required by the PDG targets cluster tightly within each branch, making the gluon-opening window a direct falsifiability handle. The two bridge factors G_d, G_u are different: they are not continuous fit parameters, but structurally fixed scale-bridge rules inside the proposed compliance-locking grammar. Their remaining open status is foundational rather than numerical: they should ultimately be derived from the same phase-stiffness functional that fixes the other interior sectors.

1 Purpose and minimal assumptions

The purpose of this document is not to reproduce the full ΦBSU field programme. It extracts one narrow question: can quark and gluon interior states be represented as a background-independent holonomy-node structure that explains fractional charge, colour singlet closures, and current-mass ratios with minimal independent inputs? The immediate precursor of the present note is the stand-alone vacuum-structure paper on the fine-structure constant and charged-lepton mass-amplitude ratios, which introduced the compressed identity/measurement grammar used here only as a minimal background vocabulary [7].

The construction uses only four minimal premises:

P1. Identity and curvature channels. A global phase identity channel carries holonomy bookkeeping, while local forces and field energy belong to a curvature channel. In the notation inherited from the ΦBSU field dictionary, this is the split

$$A = A_{\mathrm{geom}} + A_{\mathrm{id}}, \qquad A_{\mathrm{id}} = \mathrm{d}\alpha, \qquad F_{\mathrm{geom}} = \mathrm{d}A_{\mathrm{geom}}.$$

The identity channel carries phase records; the curvature channel is where a measurable interaction occurs.

P2. Physical branch selection. The operationally measured branch is denoted P_H^+. It is not assumed that every internal holonomy terminal is an asymptotic physical particle. Physical states must survive a branch projection and a closure condition.

P3. A 3^2 internal node. The quark–gluon sector is modelled on a 3^2 phase-compliance node. This is not a literal spatial lattice. It is a matrix of internal colour-tail transitions.

P4. Quadratic readout. The primary internal quantities are mass amplitudes $q_i = \sqrt{m_i}$. The observed scalar mass parameter is quadratic in the amplitude. This is the same structural reason why Koide-type relations are naturally stated in square-root masses rather than in masses themselves. A separate stand-alone note develops the fine-structure and charged-lepton version of this minimal vacuum grammar [7].

The note distinguishes three status classes. A statement called *derived* follows from the above internal grammar. A *structured hypothesis* has a clear compatibility argument but is not yet a theorem. A *scale bridge ansatz* is an empirical regularity expressed in model language and awaiting derivation.

2 The 3^2 node as a gluon-control manifold

Let the colour-tail space be

$$V_c \simeq \mathbb{C}^3.$$

The natural 3^2 object is not a set of nine unrelated slots, but the endomorphism algebra

$$\mathcal{G}_{3^2} = \mathrm{End}(V_c) = V_c \otimes V_c^*.$$

In components, a basis element

$$E^a{}_b, \qquad a, b = 1, 2, 3,$$

is a holonomy transition from colour-tail index b to colour-tail index a.

The algebra decomposes as

$$\mathrm{End}(\mathbb{C}^3) = \mathbb{C}I \oplus \mathfrak{sl}(3, \mathbb{C}).$$

In a compact operational chart the traceless part is read as $\mathfrak{su}(3)$:

$$3 \otimes \bar{3} = 1 \oplus 8.$$

This gives the central structural picture:

$$\boxed{3^2 = \text{neutral running node } (1) + \text{eight gluon-like traceless directions } (8).}$$

This is not introduced as numerology. It is the standard representation-theoretic decomposition of a colour triplet and anti-triplet. Rivero's SUSY/preon note, although not adopted here ontologically, uses the same colour decomposition for preon-antipreon pairs, $3 \times \bar{3} = 1 + 8$, and $3 \times 3 = 3 + 6$ for preon-preon pairs [5]. Here the point is different: the same algebra supplies a 3^2 holonomy-node grammar for quark terminals and gluon-like control directions.

2.1 Cartan and off-diagonal channels

The traceless sector has two qualitatively distinct kinds of directions.

- **Off-diagonal transitions** ${E^a}_b$ with $a \neq b$ change a colour-tail index. These are colour-switching holonomy moves.
- **Diagonal traceless directions** are Cartan-like compliance-pressure directions. They do not change the colour-tail index but alter relative phase pressure between colour directions.

Thus the informal language of "push and absence-bath" can be sharpened: it is a relative diagonal compliance pressure in the 3^2 node, while colour exchange is the off-diagonal holonomy transition. The identity channel records the internal holonomy; the curvature channel is the operational gluon-like interaction that appears when such holonomies are locally tested.

3 Quantum state dictionary

3.1 Quark terminals

A quark-like state is represented as a colour-indexed terminal

$$q^a \in V_c,$$

and an antiquark as a dual terminal

$$\bar{q}_a \in V_c^*.$$

A single terminal is not a closed physical state. It carries a free colour index and therefore cannot itself be a P_H^+ asymptotic particle.

We assign a triality bookkeeping number

$$\tau(q) = +1 \pmod 3, \qquad \tau(\bar{q}) = -1 \pmod 3.$$

A colour-neutral physical comparator must satisfy

$$\sum_i \tau_i = 0 \pmod 3.$$

The same framework assigns electric charge by slot orientation:

$$Q_d = -\frac{1}{3}, \qquad Q_u = +\frac{2}{3}.$$

The down-type terminal is a single negative slot orientation; the up-type terminal is a two-slot positive complement.

3.2 Gluon-like states

A gluon-like internal state is a traceless endomorphism

$$G^a{}_b \in \mathfrak{sl}(3,\mathbb{C}), \qquad \mathrm{Tr}(G) = 0.$$

The singlet part is not a ninth gluon. It is the neutral running node

$$\frac{1}{3}\delta^a_b I.$$

The gluon-like operational directions live in the traceless octet.

This distinction matters: the model does not claim an extra long-range colour singlet force. It identifies the singlet as the neutral comparator centre and the octet as the physical colour-control manifold.

3.3 Hadron closures

Observable hadrons are invariant contractions of colour-tail terminals.

Mesons are singlet contractions

$$\bar{q}_a q^a.$$

Baryons are antisymmetric three-terminal contractions

$$\epsilon_{abc} q^a q^b q^c.$$

Antibaryons use

$$\epsilon^{abc} \bar{q}_a \bar{q}_b \bar{q}_c.$$

Exotic hadrons are not counterexamples to quark non-asymptoticity. They are further invariant closures:

$$qq\bar{q}\bar{q}, \qquad qqqq\bar{q}, \qquad qqqqqq,$$

correspond respectively to tetraquark, pentaquark, and hexaquark/dibaryon-type closure patterns. Their physicality comes from total colour/triality closure, not from individual quarks becoming free particles. This agrees with the empirical QCD fact that free quarks are not observed; the Particle Data Group states that quark masses are not direct free-particle masses but parameters inferred within a theoretical framework [2, 1].

4 A π-free mass-amplitude model

The mass calculation is formulated without using an internal Euclidean angle $2\pi/3$. The symbol π belongs to a metric chart of an already dimensionalized circle. The internal quark node instead uses a cubic C_3 trace-plane invariant.

For one flavour branch $F \in \{d, u\}$, let

$$q_i^{(F)} = \sqrt{m_i^{(F)}}$$

be mass amplitudes ordered from heavy to light. We write

$$q_i^{(F)} = \bar{q}_F \left(1 + \sqrt{2}\,\widehat{\Pi}_F \, c_i(J_F)\right),$$

where

$$\sum_i c_i = 0, \qquad \sum_i c_i^2 = \frac{3}{2}.$$

The orientation is encoded by the cubic invariant

$$J_F = 4c_1c_2c_3,$$

and the three c_i are the ordered real roots of

$$4c^3 - 3c - J_F = 0.$$

The observed masses are then

$$m_i^{(F)} = G_F M_\ell \left(1 + \sqrt{2}\,\widehat{\Pi}_F\, c_i(J_F)\right)^2,$$

where

$$M_\ell = \left(\frac{\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}}{3}\right)^2$$

is used as the lepton-sector phase-stiffness reference scale. Numerically, with current charged-lepton masses,

$$M_\ell = 313.8509 \text{ MeV}.$$

The use of square-root mass amplitudes is historically tied to Koide-type relations, beginning with Koide's charged-lepton mass formula [3]. Sumino's family-gauge mechanism is a useful external example of how a 3×3 family structure can be used to protect a Koide-type relation against radiative corrections [4]. The present construction does not adopt those models. It uses the same broad lesson only in a restricted form: a three-component mass problem is most naturally written on the amplitude level and then read quadratically.

5 Cubic orientations from compliance deficits

The value $J = 1$ is a singular C_3 boundary:

$$4c^3 - 3c - 1 = (c - 1)(2c + 1)^2.$$

It has one distinguished direction and a double negative direction. The model reads quark branches as small compliance deficits away from this singular colour-node boundary:

$$J_F = 1 - \eta_F.$$

5.1 Down-type branch

The down-type terminal carries one negative electric slot,

$$Q_d = -\frac{1}{3}.$$

It is a direct one-slot deficit on the 3^2 surface. Since the measured branch is antipodally paired, the effective deficit is halved by the branch-pair factor. Thus

$$\eta_d = \frac{1}{2 \cdot 3^2},$$

and

$$\boxed{J_d = 1 - \frac{1}{2 \cdot 3^2} = \frac{17}{18}.}$$

Status: this is treated as a derived structural assignment within the proposed compliance grammar.

5.2 Up-type branch

The up-type terminal carries two positive slots,

$$Q_u = +\frac{2}{3}.$$

This is not modelled as two independent down-type deficits. It is a complementary two-slot orientation which must remain Z_3 stable under an additional triality projection. The deficit is therefore distributed through a second 3^2 layer:

$$\eta_u = \frac{2}{3^4},$$

giving

$$\boxed{J_u = 1 - \frac{2}{3^4} = \frac{79}{81}.}$$

Status: this is a structured derivation candidate. It is less direct than the down-branch rule but is not a fit to the masses; the rule is specified before the mass table is computed.

6 Gluon-manifold curvature and the down projection

The internal cycle-opening index of the gluon manifold is denoted $\widehat{\Pi}_g$. It is not the Euclidean circle constant. It is an ortho-conformal opening of the singular 3^2 comparator node.

For the up branch, the model uses a positivity-edge condition. Let $c_{\min}(J_u)$ be the most negative root of the up-branch cubic. The maximum opening before the lightest amplitude vanishes is

$$\widehat{\Pi}_{\max,u} = -\frac{1}{\sqrt{2}\, c_{\min}(J_u)}.$$

The rotating inactive antipodal pair, called the death-rider channel, leaves a non-zero edge floor

$$\epsilon_u = \frac{1}{4 \cdot 3 \cdot 3^2} = \frac{1}{108}.$$

Thus

$$\boxed{\widehat{\Pi}_g = (1 - \epsilon_u)\widehat{\Pi}_{\max,u} = 1.244671\ldots.}$$

The down branch does not experience the whole octet opening. It sees the fundamental triplet projection of the octet compliance curvature:

$$\boxed{\widehat{\Pi}_d = 1 + \frac{3}{8}(\widehat{\Pi}_g - 1) = 1.091752\ldots.}$$

The up branch uses

$$\widehat{\Pi}_u = \widehat{\Pi}_g.$$

Status: the edge equation is a closed calculation once J_u and ϵ_u are accepted. The 3/8 factor is a structured hypothesis: it is a dimension projection $\dim 3/\dim 8$, not a Casimir ratio. A Casimir projection $C_F/C_A = 4/9$ would represent interaction-strength weighting rather than compliance-opening projection and gives a poorer shape in this mass calculation.

7 Structural scale bridge and the status of G_d, G_u

The scale bridge relates the lepton phase-stiffness reference scale M_ℓ to quark current-tail mass-parameter scales. In this version the two bridge factors are not treated as continuous fit parameters. They are fixed by a discrete compliance-locking grammar associated with the $3^2 = 1 \oplus 8$ holonomy node:

$$\boxed{G_d = \frac{56}{27} = 2\left(1 + \frac{1}{3^3}\right),}$$

$$\boxed{G_u = \frac{145}{2} = 8 \cdot 3^2 + \frac{1}{2}.}$$

The remaining open question is not how to fit these values numerically, but how to derive the scale-bridge grammar from the same phase-stiffness functional that fixes the charged-lepton interior scale.

7.1 Down-branch bridge

The down branch is the direct one-slot negative surface reading associated with $Q_d = -1/3$. Its scale is carried by a two-sided antipodal tail-stiffness pair. This gives the base factor

$$G_{d,0} = 2.$$

This two-sided stiffness should not be confused with the single-branch measurement reduction $2^1/2 = 1$ used in the fine-structure threshold calculation. Here the factor 2 is the internal antipodal tail-pair stiffness that remains active for a colour-node endpoint.

Because a quark endpoint is not a closed lepton-like interior but a partial terminal of the 3^2 node, a minimal full-$3R$ bath-locking residual remains. The residual fraction is

$$3^{-3},$$

acting on the whole antipodal tail pair. Hence

$$G_d = 2\left(1 + \frac{1}{3^3}\right) = \frac{56}{27}.$$

This is the scale analogue of the down-branch orientation deficit

$$J_d = 1 - \frac{1}{2 \cdot 3^2},$$

but it should not be conflated with it. The J_d deficit fixes the direction of the branch in the C_3 trace plane, while G_d fixes the overall tail-stiffness scale relative to M_ℓ.

7.2 Up-branch bridge

The up branch is the two-slot positive complement associated with $Q_u = +2/3$. It is not the sum of two down-type slots. Its bulk scale uses the full traceless gluon octet together with the 3^2 surface-of-surface compliance projection:

$$G_{u,0} = 8 \cdot 3^2 = 72.$$

The near-edge up-complement also carries a one-branch death-rider threshold term. This contribution is a boundary term and is not replicated over the octet, giving

$$G_u = 8 \cdot 3^2 + \frac{1}{2} = \frac{145}{2}.$$

Again, this is distinct from the up-branch orientation deficit

$$J_u = 1 - \frac{2}{3^4}.$$

The J_u deficit fixes the complement orientation in the C_3 trace plane, the death-rider floor fixes the edge opening, and G_u fixes the branch scale.

7.3 Status of the bridge

The scale bridges are therefore fixed once four discrete rules are accepted: the $3^2 = 1 \oplus 8$ node decomposition, the antipodal tail-stiffness rule, the full-$3R$ bath-lock residual, and the death-rider boundary term. They are not freely adjusted continuous parameters. Their remaining open status is foundational: the bridge rules should ultimately be obtained from the same κ_Φ phase-stiffness functional that gives the lepton interior scale.

The calculation below is therefore split into two claims. The branch-internal shape ratios are independent of G_d, G_u and provide the most direct falsifiability test. The absolute current-tail mass-parameter table uses the structural bridge rules above. If those bridge rules are later derived from κ_Φ, the six-entry table becomes a constrained prediction; if not, the state-compatibility and scale-free shape layers remain the more fundamental content of the note.

8 Numerical calculation

The following numbers are computed from

$$J_d = 17/18, \quad J_u = 79/81, \quad \widehat{\Pi}_g = 1.244671\ldots, \quad \widehat{\Pi}_d = 1 + \frac{3}{8}(\widehat{\Pi}_g - 1),$$

and scale bridge values

$$G_d = 56/27, \qquad G_u = 145/2.$$

The roots of the cubic are

$$c^{(d)} = (0.993776, -0.400412, -0.593363),$$

$$c^{(u)} = (0.997246, -0.434399, -0.562847),$$

where entries are ordered as heavy, middle, light.

Table 1: Predicted current-tail mass parameters compared with PDG 2025 consensus mass-parameter targets. The target types are not identical observables: u, d, s are light $\overline{\text{MS}}$ current-mass proxies at 2 GeV; c, b are heavy $\overline{\text{MS}}$ mass proxies at their own mass scales; t is an event-kinematic proxy. The residual column is a percentage deviation from the PDG consensus target, not a statistical significance or a unified sigma measure.

State	target type	model [MeV]	PDG target [MeV]	residual
u	light current proxy	1.951	2.16	-9.68%
d	light current proxy	4.578	4.70	-2.59%
s	light current proxy	94.878	93.5	$+1.47\%$
c	heavy current proxy	1260.416	1273.0	-0.99%
b	heavy current proxy	4181.039	4183.0	-0.047%
t	event-kinematic proxy	172753.1	172560.0	$+0.112\%$

Table 2: Scale-free shape tests. These ratios depend on $J_d, J_u, \widehat{\Pi}_g$ and the 3/8 projection, not on G_d, G_u.

Ratio	model	PDG target ratio	relative difference
m_b/m_s	44.068	44.738	−1.50%
m_s/m_d	20.723	19.894	+4.17%
m_t/m_c	137.060	135.554	+1.11%
m_c/m_u	646.101	589.352	+9.63%

The within-branch ratios are independent of the scale bridge factors G_d, G_u:

The large residual in m_c/m_u is not hidden: the lightest up-terminal sits closest to the positivity edge and is consequently most sensitive to small tail-compliance wobble or to the operational definition of the light current-mass proxy. In the model, the up-light amplitude is only a small floor above zero; therefore a small change in the denominator-side tail or in the operational light-quark proxy produces a disproportionately large change in m_c/m_u. This is not an excuse to ignore the discrepancy. It identifies the light up state as the first place where the core lock should either fail or require a controlled tail-wobble correction.

9 Effective curvature consistency check

A useful internal check is to invert the formula and ask what opening $\widehat{\Pi}_i$ each PDG target would require after J_F and G_F have been fixed. This gives

$$\widehat{\Pi}_i^{\text{obs}} = \frac{\sqrt{m_i/(G_F M_\ell)} - 1}{\sqrt{2}\, c_i}.$$

For the down branch the values are approximately

$$\widehat{\Pi}_d^{\text{obs}}(d) = 1.0904, \quad \widehat{\Pi}_d^{\text{obs}}(s) = 1.0967, \quad \widehat{\Pi}_d^{\text{obs}}(b) = 1.0922.$$

The predicted value is

$$\widehat{\Pi}_d = 1.0918.$$

For the up branch,

$$\widehat{\Pi}_u^{\text{obs}}(u) = 1.2441, \quad \widehat{\Pi}_u^{\text{obs}}(c) = 1.2428, \quad \widehat{\Pi}_u^{\text{obs}}(t) = 1.2436,$$

while

$$\widehat{\Pi}_u = \widehat{\Pi}_g = 1.2447.$$

This check is more informative than the raw mass table. It shows that, after the scale bridge, the required gluon-manifold opening is nearly constant within each branch. The residuals then look like tail-wobble corrections around a stable branch lock, not arbitrary independent mass errors.

9.1 How narrow is the gluon-opening window?

The same calculation gives a quantitative falsifiability window for $\widehat{\Pi}_g$. Keeping $J_d = 17/18$, $J_u = 79/81$ and the 3/8 projection fixed, one may vary only $\widehat{\Pi}_g$ and recompute the four scale-free ratios in Table 2. The result is sharply constrained by the light up ratio m_c/m_u, because the up-light amplitude is close to the positivity edge.

Table 3: Sensitivity of the scale-free ratios to the common gluon-opening index $\widehat{\Pi}_g$. The intervals list values of $\widehat{\Pi}_g$ for which all four ratios in Table 2 remain within the stated relative tolerance of the PDG consensus target ratios.

Criterion	lower $\widehat{\Pi}_g$	upper $\widehat{\Pi}_g$	width
all four ratios within 10%	1.24342	1.24469	0.00127
all four ratios within 5%	1.24378	1.24441	0.00063

The edge-derived value used in this note is

$$\widehat{\Pi}_g = 1.244671\ldots,$$

which lies inside the 10% all-ratio window but just outside the 5% all-ratio window, due almost entirely to m_c/m_u. If the light up ratio is excluded as a known edge-sensitive tail state, the remaining three ratios allow a much broader 5% interval of approximately

$$1.2346 \lesssim \widehat{\Pi}_g \lesssim 1.2459.$$

Thus the model is not protected by a broad continuous freedom: the full four-ratio test forces a narrow opening window, and the most sensitive external handle is precisely the light up branch.

10 Parameter freedom and falsifiability

It is useful to separate three claims that are often conflated in informal discussions: quantum-state compatibility, scale-free mass shape, and absolute branch scale.

10.1 Quantum-state compatibility layer

The quantum-state layer is independent of the mass-table fit. It follows from the 3^2 node and its invariant contractions:

$$3 \otimes \bar{3} = 1 \oplus 8,$$

$$q^a \in \mathbb{C}^3, \qquad \bar{q}_a \in (\mathbb{C}^3)^*,$$

$$\bar{q}_a q^a \quad \text{for mesons}, \qquad \epsilon_{abc} q^a q^b q^c \quad \text{for baryons}.$$

The same triality rule covers exotic hadronic closures such as $qq\bar{q}\bar{q}$, $qqqq\bar{q}$, and $qqqqqq$. In this sense the fractional charge and colour-index bookkeeping are not fit parameters. The hypothesis derives the compatibility conditions for quark-like terminals and hadronic physical states before any mass values are inserted.

10.2 Scale-free mass-shape layer

The scale-free mass-shape layer is specified by

$$J_d = 17/18, \qquad J_u = 79/81, \qquad \widehat{\Pi}_g = (1 - 1/108)\widehat{\Pi}_{\max,u}, \qquad \widehat{\Pi}_d = 1 + \frac{3}{8}(\widehat{\Pi}_g - 1).$$

These quantities determine the branch-internal ratios in Table 2. They do not use the overall scale bridge factors G_d or G_u.

Thus the present mass calculation has a derived, falsifiable part: the four independent within-branch ratios

$$m_b/m_s, \quad m_s/m_d, \quad m_t/m_c, \quad m_c/m_u$$

come from the C_3 shape and gluon-opening hypotheses. The six quark current-tail entries in Table 1 then follow once two branch scales are supplied. The quantum-state compatibility and the mass-shape ratios are therefore more constrained than the absolute table.

10.3 Scale bridge layer

The absolute branch scales require

$$G_d = 56/27, \qquad G_u = 145/2.$$

In the present version these are treated as *structural scale-bridge rules*, not as continuous fit parameters. The proposed rules are

$$G_d = 2\left(1 + \frac{1}{3^3}\right),$$

$$G_u = 8 \cdot 3^2 + \frac{1}{2}.$$

The first term in G_d is the two-sided antipodal tail-stiffness pair; the 3^{-3} term is the minimal full-$3R$ bath-locking residual. The leading term in G_u is the full traceless octet times the 3^2 surface-of-surface compliance projection; the 1/2 term is a one-branch death-rider threshold contribution. These rules are discrete and fixed once the compliance-locking grammar is accepted. What remains to be done is to derive that grammar from the same phase stiffness κ_Φ that fixes the lepton interior scale.

The effective parameter-freedom status is therefore the following.

Layer	quantity	status
Quantum states	fractional charge; triality; δ, ϵ closures	derived inside the node hypothesis
Shape	$J_d = 17/18$	derived in compliance grammar
Shape	$J_u = 79/81$	structured derivation candidate
Opening	$\widehat{\Pi}_g = (1 - 1/108)\widehat{\Pi}_{\max,u}$	calculated once J_u and the death-rider floor are accepted
Opening	$\widehat{\Pi}_d = 1 + (3/8)(\widehat{\Pi}_g - 1)$	dimension-projection hypothesis
Scale	$G_d = 56/27$	structurally fixed bridge rule; variational derivation still open
Scale	$G_u = 145/2$	structurally fixed bridge rule; variational derivation still open

Consequently the model contains no continuous quark-mass fit parameters once the discrete bridge rules are accepted. The hard content is the quantum-state compatibility, the scale-free branch shape, and the fixed bridge grammar above. The remaining incompleteness is not numerical but foundational: a full derivation should show how the G_d and G_u bridge rules descend from the universal phase stiffness κ_Φ.

A strong failure would occur if improved consensus mass targets destroyed the scale-free ratios of Table 2 or if the 3^2 node failed to reproduce the colour/triality closure structure. A deeper failure would occur if no principled route to G_d and G_u could be found from κ_Φ and the gluon-compliance manifold.

11 Relation to standard QCD and to ΦBSU

This construction is not presented as a replacement for operational QCD. The standard gauge description, hadron spectroscopy, lattice calculations, and collider extractions remain the operational framework that defines the comparison targets. The present claim is more limited: the 3^2 holonomy node may provide a background-independent interior substrate whose operational chart is the familiar colour-triplet and gluon-octet structure.

The relation to the ΦBSU framework is similarly limited. The model uses the ΦBSU identity/curvature split and the idea that a measured scalar is a quadratic readout of a projective amplitude. It does not require the reader to accept the entire cosmological or gravitational programme. The quark-gluon proposal stands or falls on whether the 3^2 node, the invariant closures, and the C_3 mass-amplitude layer remain predictive.

12 Open problems

The open problems are ordered here by near-term falsifiability rather than by conceptual depth.

1. **Collider and heavy-flavour diagnostics.** If $\widehat{\Pi}_g$ is a real gluon-manifold opening, traces may appear in heavy-flavour fragmentation, jet substructure, top phenomenology, and heavy-hadron form factors. The narrow $\widehat{\Pi}_g$ window above gives this problem immediate diagnostic value: heavy-flavour data could support, move, or destroy the required opening window without waiting for the full scale-bridge derivation.

2. **Hadron energy.** The model concerns current-tail mass parameters. Hadron masses require the energy of the closed Z_3 comparator, including gluon bath and anomaly-like terms. In QCD, most of the proton mass is dynamical rather than explicit light-quark current mass, so hadron closure must be treated separately.

3. **Tail-wobble corrections.** The residuals, especially in the light up branch, suggest additive wobble in the denominator D_i, not a change in the core lock. A controlled tail-wobble algebra should distinguish Cartan-like mass corrections from off-diagonal mixing corrections.

4. **Derive the scale-bridge grammar from κ_Φ.** The discrete bridge rules $G_d = 56/27$ and $G_u = 145/2$ are structurally fixed inside the proposed compliance grammar, but their deepest justification still requires a derivation from κ_Φ and the gluon-compliance manifold. Until this is done, the absolute scale layer remains conditional on the bridge grammar, whereas the quantum-state and scale-free shape layers are the firmer content.

13 Conclusion

The quark-gluon sector can be represented compactly by a 3^2 holonomy node. The decomposition

$$3 \otimes \bar{3} = 1 \oplus 8$$

provides a neutral running node plus an eight-direction gluon-like manifold. Quarks are colour-indexed partial terminals, not closed asymptotic particles; hadrons are invariant closures through δ^a_b, ϵ_{abc}, and their composites. Fractional charges arise as slot orientations, while total physical states require integer/triality-neutral closure.

For the mass-amplitude sector, the most productive present formulation is π-free: a cubic C_3 invariant J_F fixes branch orientation, and an internal opening $\widehat{\Pi}_F$ measures how far the singular colour node has been opened into an effective dimensional structure. The scale-free predictions are already nontrivial. The scale bridge to absolute current-tail mass parameters is strengthened by the discrete bridge rules $G_d = 56/27$ and $G_u = 145/2$. These are not continuous fit parameters; they are structurally fixed once the proposed compliance-locking grammar is accepted, although their final derivation from the universal phase-stiffness functional remains open.

The immediate research target is therefore twofold. First, the narrow $\widehat{\Pi}_g$ window and the near-constant inverted openings within each branch should be confronted with heavy-flavour and hadronization data. Second, the discrete scale-bridge grammar should be derived from κ_Φ and the $3^2 = 1 \oplus 8$ gluon-compliance manifold, while keeping tail-wobble corrections separate from the core C_3 lock. Until that variational derivation is complete, the model should be read as a structured and testable quark-interior hypothesis whose state-compatibility, scale-free mass-ratio layer, and discrete bridge rules are fixed, while the deeper origin of the bridge grammar remains open.

References

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