ΦBSU RESEARCH ESSAY

One-Loop Hypersymmetry

in the ΦBSU Unification Programme

A theorem-first synthesis of phase-current pairing, premetric Wistor dynamics, emergent gauge geometry, and the unification horizon

Purpose. This document is a curiosity-driven research synthesis rather than a claim of completed unification. It places the hypersymmetry idea in relation to established mathematical physics, separates proved statements from model-specific constructions and conditional claims, and identifies the next calculations that can decide whether the programme has physical depth.

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Independent research essay in the ΦBSU programme

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Example synthesis based on the current Part I, Part II, hypersymmetry, vacuum-structure, quark–gluon, and Woit discussion materials.

Tiivistelmä

Tässä tutkielmassa hypersymmetria muotoillaan uudelleen ilman fundamentaalista peiliä, kätisyyttä tai erillistä partnerihiukkasten sektoria. Perusrakenne on taustametriikasta riippumaton naapuruuskompleksi, jossa solmuvaiheen fluktuaatio $\eta \in C^0$ tuottaa tarkan naapuruusvirran $D\eta \in C^1$. Kvadratiiviset operaattorit

$$H_0 = D^\dagger D, \qquad H_1 = DD^\dagger$$

ovat nollasta poikkeavalla sektorilla isospektraalisia. Tämä on dokumentin varsinainen tarkistuskelpoinen hypersymmetriatulos. Se ei vielä yksin todista fysikaalisen efektiivisen toiminnon kumoutumista: vastakkaisen determinanttipainon on synnyttävä eksplisiittisestä rajoitemitasta, Jacobista tai BRST/BV-kompleksista.

Tutkielma sijoittaa tuloksen suhteessa gauge-teoriaan, Hodge-teoriaan, supersymmetriaan, analyyttiseen torsioon, Osterwalder–Schrader-rekonstruktioon ja twistoritutkimukseen. ΦBSU:n oma uutuus on tulkinta: paritetut sektorit eivät ole bosoni ja fermioni tai fysikaalinen maailma ja peilimaailma, vaan saman paikallisen vaihepäivityksen potentiaali- ja virtalukutavat. Globaalit poikkeukset jäävät nollamoodeihin, kohomologiaan ja K_2-palautustopologiaan.

Mukana on kaksi äärellistä, suoraan tarkistettavaa mallia. Painotetussa neljän solmun ja viiden särmän verkossa H_0:n ja H_1:n kolme positiivista ominaisarvoa yhtyvät numeerisesti, kun taas virtasektorille jää yksi ylimääräinen syklinen nollamoodi. Klein-pullon yksinkertaisessa twisted-paikallissysteemissä $\rho(a) = 1$ ja $\rho(b) = -1$ ketjukompleksi on asyklinen, mikä osoittaa konkreettisesti, että seam-holonomia voi poistaa topologisia nollamoodeja ilman fysikaalista peilisektoria.

Taustariippumattomuus luetaan kerroksittain. Fundamentaalilla tasolla ovat naapuruus, vaihe-erot, paikallisesti säilyvät vaihevirrat ja jäyhyysfunktionaali κ_Φ. Lorentzinen mittarimonisto M_4, spinorikätisyys ja CPT syntyvät vasta koherentin operatiivisen kenttäalgebran yhteydessä. Euklidisointi kuuluu ainoastaan epävarmuus-paikalliseen one-loop-regulaatioon emergentin M_4-kartaston sisällä. Näin Peter Woitin euklidis-holomorfinen rekonstruktio näyttäytyy hyödyllisenä myöhemmän vaiheen varoituksena, ei Wistor-moottorin ontologisena lähtökohtana.

Lopuksi tutkielma erottaa unifikaatio-ohjelman tasot: (i) todistettu spektriparitus, (ii) eksplisiittiset ΦBSU-rakenteet kuten identiteetti/kaarevuus-jako ja K_2-palautus, (iii) ehdollinen one-loop-kumoutuminen sekä (iv) avoin κ_Φ-johto, emergentti M_4, gauge- ja aine-esitysten yhteinen alkuperä sekä ristimittakaavainen fenomenologia. Dokumentin tarkoitus on näyttää, millä tavoin kurkottava unifikaatiotavoite voi pysyä arvokkaana vain, jos sen alimmat portaat ovat itsenäisesti tarkistettavia.

Lukijalle

Dokumentissa sanat *teoreema*, *eksplisiittinen konstruktio*, *ehdollinen väite* ja *avoin tavoite* pidetään tarkoituksella erillään. Unifikaatiokuva ei ole todiste alimman tason rakenteille; sen sijaan alemman tason rakenteiden on kestettävä tarkistus riippumatta siitä, toteutuuko laaja unifikaatiotulkinta.

Abstract

This essay reformulates ΦBSU hypersymmetry as a one-loop phase–current pairing on a premetric neighbourhood complex. The primitive quadratic map is

$$D = K^{1/2} d_0 : C^0(\mathcal{C}) \longrightarrow C^1(\mathcal{C}),$$

where d_0 is the neighbourhood coboundary and K is the linearized phase-stiffness response. The operators $H_0 = D^\dagger D$ and $H_1 = DD^\dagger$ have identical non-zero spectra. Their paired heat-kernel contributions cancel in a graded trace, while zero modes retain global phase, cycle, seam, and twisted-cohomology information. This theorem is standard operator mathematics; the ΦBSU novelty is the physical interpretation of the two sides as a phase fluctuation and its exact neighbour current, with no fundamental mirror world, chirality flip, or additional asymptotic particle.

The essay distinguishes spectral pairing from physical determinant cancellation. A cancellation in the effective action requires a separately derived functional measure, constraint Jacobian, or BRST/BV superdeterminant. Two finite calculations are supplied: a weighted graph with explicit paired spectra and a one-dimensional local system on the Klein bottle whose twisted cellular complex is acyclic. These make the bulk pairing and topological residue directly auditable.

The broader synthesis places the construction on the shoulders of gauge theory, Hodge theory, supersymmetric quantum mechanics, analytic torsion, Osterwalder–Schrader reconstruction, and twistor geometry. Euclideanization is restricted to an uncertainty-local regulator patch after an operational Lorentzian M_4 chart has emerged. Fundamental chirality and CPT are not assumed at the premetric level. The final sections organise the unification programme into proved results, explicit model structures, conditional one-loop statements, and open targets: the derivation of the universal stiffness functional κ_Φ, the emergence of the metric and causal chart, the physical measure, anomaly control, and the matching to gauge, matter, and cross-scale phenomenology.

Keywords: hypersymmetry; one-loop spectral pairing; cochain complex; background independence; Wistor; Klein bottle; emergent gauge geometry; ΦBSU.

Contents

1 Why a synthesis rather than another narrow claim

The most useful form of a curiosity-driven unification programme is neither a manifesto nor a sequence of isolated numerical coincidences. It is a layered research object. At the bottom must be statements that can be checked without accepting the whole ontology. Above them may sit interpretations, matching rules, and speculative bridges. The unification horizon is then allowed to be ambitious precisely because the route toward it is divided into independently vulnerable steps.

The original hypersymmetry note was intentionally exploratory. It presented a pseudo-mirror sheet, an antilinear reflection R, a projection P_+, determinant differences, and anomaly cancellation as a compact provisional language for partnerless one-loop stabilization [1]. Subsequent discussion exposed a cleaner mathematical core. The stable content is not a literal reflection symmetry. It is the pairing of a phase fluctuation with the exact current generated by that fluctuation. The reflection language can be retained as historical motivation or as an operational chart metaphor, but it should not carry the formal proof.

This essay therefore serves four purposes.

1. It identifies the inherited paradigm: relativity, gauge connections, holonomy, one-loop determinants, Hodge complexes, supersymmetric pairing, analytic torsion, and reconstruction between Euclidean and Lorentzian descriptions.
2. It isolates the checkable mathematical result that deserves the name hypersymmetry in the present programme.
3. It explains which additional assumptions are required before spectral pairing becomes physical vacuum-energy or beta-function cancellation.
4. It reconnects the result to the larger ΦBSU ambition without confusing a unification map with a completed derivation.

Reading rule

The paper is intentionally generous toward exploratory ideas but strict about status. A result can be interesting before it is complete. What it cannot be is simultaneously advertised as a theorem and protected from the tests appropriate to a theorem.

2 The inherited paradigm: shoulders on which the proposal stands

2.1 Relativity, physical spacetime, and background independence

General relativity replaces a fixed gravitational force field by dynamical geometry. Yet a claim of background independence can mean several different things. It may mean diffeomorphism covariance on a manifold already supplied with dimension and signature; it may mean that a metric is dynamical rather than fixed; or, more strongly, it may mean that even the operational manifold and its causal signature emerge from more primitive relations.

The ΦBSU programme aims at the strongest reading. Its Part I language treats invariant separations as primary and coordinates and metrics as bookkeeping devices. The operational dictionary introduces a global phase $\Phi = \mathrm{e}^{\mathrm{i}\alpha}$, a density $\rho = \|\nabla\alpha\|$, and a buoyancy field $a_\mu = -\partial_\mu \ln\rho$ [2]. That dictionary is useful on an M_4 chart, but it does not by itself prove strict background independence because the norm and derivative already presuppose an operational geometric structure. The present synthesis therefore distinguishes the premetric Wistor layer from its M_4 realization.

2.2 Gauge connections, curvature, and holonomy

The gauge-theoretic distinction most important for the programme is the distinction between local curvature and global holonomy. For a $U(1)$ phase one may write

$$A = A_{\mathrm{geom}} + A_{\mathrm{id}}, \qquad A_{\mathrm{id}} = -\mathrm{i}\Phi^{-1}\mathrm{d}\Phi = \mathrm{d}\alpha, \qquad F_{\mathrm{geom}} = \mathrm{d}A_{\mathrm{geom}}. \tag{1}$$

In a smooth simply connected patch, $\mathrm{d}A_{\mathrm{id}} = 0$. Nevertheless, a closed comparator may retain a non-trivial integral of A_{id} when the phase is globally multi-valued or the domain contains defects. This is the same broad lesson that makes the Aharonov–Bohm effect conceptually important: local curvature and global phase comparison are not identical physical data [8, 9].

In ΦBSU this split is not merely a convenient decomposition of a known gauge field. It is intended as a separation between identity bookkeeping and locally force-carrying geometry. The present essay uses it as an explicit model construction, not as an already established derivation of electromagnetism.

2.3 One-loop effective actions and spectral data

For a quadratic bosonic fluctuation operator Δ, the formal one-loop contribution has the form

$$\Gamma^{(1)}_{\mathrm{bos}} = \frac{1}{2}\,\mathrm{Tr}\ln\Delta, \tag{2}$$

while Grassmann integration produces an opposite sign. Regularized determinants are often analysed through the heat kernel,

$$\ln\det{}'\Delta = -\int_0^\infty \frac{\mathrm{d}t}{t}\,\mathrm{Tr}'\mathrm{e}^{-t\Delta}, \tag{3}$$

with the prime excluding zero modes. The ultraviolet content is encoded in the small-t asymptotics; global phases and zero modes require additional treatment. Heat kernels, zeta determinants, and index theory are therefore a natural language for any one-loop stabilization proposal [14, 15].

2.4 Supersymmetry, Hodge theory, and cohomological pairing

Supersymmetric quantum mechanics made a profound connection between a supercharge and the de Rham complex. With

$$Q = \mathrm{d}, \qquad Q^\dagger = \delta, \qquad H = QQ^\dagger + Q^\dagger Q,$$

non-zero modes pair between adjacent form degrees, while harmonic forms remain unpaired and encode topology. Witten used this structure to relate supersymmetry, Morse theory, and topology [11]. The same operator skeleton underlies Hodge decomposition, analytic torsion, and many cohomological field theories [12, 13].

The mathematical pairing

$$D^\dagger D \quad \leftrightarrow \quad DD^\dagger$$

is therefore not new. The potentially new element in the present programme is the physical interpretation: the two paired objects are not assumed to be particles of opposite statistics or members of a super-Poincare multiplet. They are a phase fluctuation and the exact neighbour current generated by it.

2.5 Twistors, Wick rotation, and reconstruction

Twistor theory reorganizes four-dimensional conformal and massless-field geometry in terms of spinorial and holomorphic data [20, 21]. Peter Woit's 2026 draft emphasizes that a purely chiral holomorphic theory is not naturally recovered by naively complexifying spacetime and reversing a Wick rotation. He instead highlights conjugation and Osterwalder–Schrader-type reconstruction as a route from Euclidean holomorphic data to a Minkowski state space [6, 18, 19].

That analysis is valuable, but it begins after several structures have already been chosen: Euclideanity, holomorphy, chirality, and a twistor ambient. The Wistor engine developed here begins earlier. It treats neighbourhood and phase-current compatibility as primitive; Lorentzian and Euclidean charts appear only later. This distinction becomes important in section 8.

3 The ΦBSU starting point: a premetric Wistor engine

3.1 Wistor is not original twistor space

The term *Wistor* is used here for the premetric neighbourhood phase-current engine of ΦBSU. It should not be identified with Penrose twistor space $\mathbb{CP}^3$. Ordinary twistor variables may become an efficient chart of an emergent null geometry, but they are not assumed as the primitive ontology.

Let $\mathcal{C}$ be a finite or locally finite oriented cell complex used as an auxiliary representation of an unoriented neighbourhood structure. Its cochain sequence begins

$$C^0(\mathcal{C}) \xrightarrow{d_0} C^1(\mathcal{C}) \xrightarrow{d_1} C^2(\mathcal{C}), \qquad d_1 d_0 = 0. \tag{4}$$

A phase configuration is $\alpha \in C^0$. Its neighbour difference is $d_0\alpha \in C^1$. A positive edge-stiffness operator K defines the linearized current

$$J = K d_0 \alpha. \tag{5}$$

The adjoint $\delta_0 = d_0^\dagger$ supplies a divergence. A source-free local balance law is

$$\delta_0 J = 0. \tag{6}$$

The orientation of each edge is computational. Reversing an edge changes the signs of both its phase difference and current. It does not create a new physical direction or a mirror event.

3.2 Local mediation and globally non-local invariants

For a set of vertices Ω, summing equation (6) over Ω cancels every internal edge twice with opposite orientation. Only the boundary flux remains. This is the discrete Stokes principle. The dynamics is neighbour-mediated, yet the integrated invariant can depend on an extended region or a closed loop.

A closed comparator γ carries the holonomy

$$\mathrm{Hol}_\gamma(\alpha) = \exp\left(\mathrm{i}\sum_{e\in\gamma}(d_0\alpha)_e\right). \tag{7}$$

The loop result is non-local as an invariant, not as a superluminal influence. It becomes operational only when locally propagated histories are compared after closure. This is the discrete form of Part I's stated locality rule [2].

3.3 Premetric density and buoyancy as a proposed coarse variable

A metric-free local phase-intensity proxy can be defined at a vertex v by

$$\rho_v^2 = \sum_{e\in\mathrm{star}(v)} \omega_{ve}\,(d_0\alpha)_e^2, \tag{8}$$

where the positive weights ω_{ve} belong to the stiffness and adjacency data, not to a pre-given spacetime norm. A neighbour buoyancy difference is then

$$a_e = -(d_0 \ln \rho)_e. \tag{9}$$

Explicit ΦBSU construction

Equations (8)–(9) are a premetric candidate for the continuum dictionary $\rho = \|\nabla\alpha\|$ and $a_\mu = -\partial_\mu \ln \rho$. Their merit is that they do not require an external metric at the primitive level. Their limitation is that the continuum scaling, covariance, and causal signature remain to be derived from κ_Φ.

3.4 From $3T \times 3R$ to an operational $M_4 \times K_2$ chart

The wider programme interprets the operational $M_4 \times K_2$ description as a measurable chart of a primary $3T \times 3R$ separability structure [1, 3]. In the present synthesis this is a programme-level map, not a theorem:

$$(3T \times 3R, \alpha, \kappa_\Phi) \longrightarrow \text{coherent causal chart } (M_4, g_{\mu\nu}) \quad + \text{return topology } K_2. \tag{10}$$

The K_2 factor is not interpreted as a second physical world. It is a compact bookkeeping device for non-orientable return relations, twisted boundary conditions, and seam holonomy. A concrete local-system calculation appears in section 7.4.

4 Hypersymmetry redefined

4.1 What the term does not mean

At the premetric level there is no physical parity operation, no left/right spinor decomposition, no sign of rotation, and no time-reversal map. Those require an oriented metric chart and a local field algebra. Accordingly, the present use of *hypersymmetry* does not mean

- a fundamental mirror sheet,
- a second Hilbert space of hidden particles,
- an antilinear grading of a complex state space,
- a super-Poincare algebra,
- or an all-orders ultraviolet completion.

The term refers narrowly to a one-loop pairing built into the phase–current complex.

4.2 Quadratic phase stiffness

Let α_0 be a background configuration and $\eta \in C^0$ a fluctuation. Assume only that the unknown universal functional admits a quadratic expansion on the stable patch,

$$S_\Phi[\alpha_0 + \eta] = S_\Phi[\alpha_0] + \frac{1}{2}\langle d_0\eta, K_{\alpha_0} d_0\eta\rangle + O(\eta^3), \tag{11}$$

with K_{α_0} positive and self-adjoint on the regulated local sector. Define

$$D := K_{\alpha_0}^{1/2} d_0, \qquad H_0 := D^\dagger D, \qquad H_1 := DD^\dagger. \tag{12}$$

H_0 acts on phase fluctuations. H_1 acts on exact current amplitudes in $\operatorname{im} D \subset C^1$ and extends naturally to all one-cochains.

Definition 4.1 (One-loop phase–current hypersymmetry). *The one-loop hypersymmetry of a stable Wistor patch is the non-zero spectral pairing of $H_0 = D^\dagger D$ and $H_1 = DD^\dagger$, together with the interpretation of their unpaired kernels as global phase, loop, seam, or twisted-cohomology data.*

4.3 No fundamental sign: orientation invariance

Proposition 4.2 (Auxiliary edge orientation does not affect the phase spectrum). *Let $S : C^1 \to C^1$ be diagonal with entries ± 1, representing arbitrary reversal of edge orientations, and define $D' = SD$. Then*

$$H_0' = D'^\dagger D' = H_0, \qquad H_1' = D'D'^\dagger = SH_1S^{-1}. \tag{13}$$

Therefore H_0 is unchanged and H_1 is unitarily equivalent to its original form.

Proof. Since $S^\dagger S = I$ and $S^{-1} = S$, one has $D'^\dagger D' = D^\dagger S^\dagger S D = D^\dagger D$. Likewise $D'D'^\dagger = SDD^\dagger S$. Hence the spectra and all spectral invariants are orientation-independent. □

Established result

This proposition formalizes the statement that plus/minus edge direction is a coordinate convention. It is not a microscopic handedness. Chirality may still emerge later in an oriented M_4 spin representation, but it is not present in the premetric pairing theorem.

5 The main theorem: phase–current spectral pairing

Theorem 5.1 (Non-zero isospectrality). *Let $D : \mathcal{H}_0 \to \mathcal{H}_1$ be a densely defined closed operator between Hilbert spaces, with $H_0 = D^\dagger D$ and $H_1 = DD^\dagger$. Then the non-zero spectra of H_0 and H_1 agree, including multiplicities:*

$$\operatorname{spec}'(H_0) = \operatorname{spec}'(H_1). \tag{14}$$

For every eigenvector u of H_0 with eigenvalue $\lambda > 0$, the vector $\lambda^{-1/2}Du$ is an eigenvector of H_1 with the same eigenvalue. The inverse map is $v \mapsto \lambda^{-1/2}D^\dagger v$.

Proof. If $D^\dagger Du = \lambda u$ with $\lambda > 0$, then $Du \neq 0$ and

$$DD^\dagger(Du) = D(D^\dagger D)u = \lambda Du.$$

Conversely, if $DD^\dagger v = \lambda v$, then $D^\dagger v \neq 0$ and

$$D^\dagger D(D^\dagger v) = D^\dagger(DD^\dagger)v = \lambda D^\dagger v.$$

The normalized maps are inverse isometries on each positive eigenspace. The continuous-spectrum statement follows from the polar decomposition of D. □

Corollary 5.2 (Paired functional calculus). *For any suitable Borel function f for which the traces exist,*

$$\operatorname{Tr}' f(H_0) = \operatorname{Tr}' f(H_1). \tag{15}$$

In particular,

$$\det{}' H_0 = \det{}' H_1, \qquad \operatorname{Tr}' \mathrm{e}^{-tH_0} = \operatorname{Tr}' \mathrm{e}^{-tH_1}. \tag{16}$$

5.1 Cochain Dirac operator and index residue

Introduce

$$\mathcal{D}_{\mathcal{N}} = \begin{pmatrix} 0 & D^\dagger \\ D & 0 \end{pmatrix}, \qquad \Gamma = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \tag{17}$$

Then $\{\Gamma, \mathcal{D}_{\mathcal{N}}\} = 0$ and

$$\mathcal{D}_{\mathcal{N}}^2 = \begin{pmatrix} H_0 & 0 \\ 0 & H_1 \end{pmatrix}. \tag{18}$$

The grading Γ is complex-linear. It is a cochain-degree grading, not a physical reflection. The heat supertrace is

$$\operatorname{Str} \mathrm{e}^{-t\mathcal{D}_{\mathcal{N}}^2} = \operatorname{Tr} \mathrm{e}^{-tH_0} - \operatorname{Tr} \mathrm{e}^{-tH_1} = \dim\ker D - \dim\ker D^\dagger = \operatorname{ind} D. \tag{19}$$

Every non-zero mode cancels. Only the index remains.

5.2 Hodge decomposition and the meaning of unpaired modes

On a finite cell complex with the full sequence (4), one has the discrete Hodge decomposition

$$C^1 = \operatorname{im} d_0 \oplus \mathcal{H}^1 \oplus \operatorname{im} d_1^\dagger, \tag{20}$$

where $\mathcal{H}^1 \simeq H^1(\mathcal{C})$ is the harmonic cycle sector. The exact current $D\eta$ lives in the first summand. Closed non-exact currents live in $\mathcal{H}^1$ and survive as global loop data. Coexact currents belong to a higher constraint sector and are not generated by a scalar phase alone.

Established result

The phase–current pairing does not erase topology. It cancels the non-zero exact bulk spectrum. Global phase modes, independent cycles, boundary modes, and twisted seam classes remain precisely where cohomology says they should remain.

6 What the theorem does not yet prove: the functional measure

6.1 Isospectrality is not a minus sign

The equality $\det{}' H_0 = \det{}' H_1$ does not imply

$$\frac{1}{2}\ln\det{}' H_0 - \frac{1}{2}\ln\det{}' H_1 = 0$$

unless the physical path integral actually assigns opposite determinant powers to the two sectors. Two ordinary bosonic integrations give a sum, not a difference. The original reflection draft inserted the difference as part of the proposed mechanism [1]; the revised formulation makes the missing step explicit.

Conditional statement

A partnerless one-loop cancellation follows if the exact-current sector enters as a constraint, Jacobian, ghost, or cohomological auxiliary sector with the opposite determinant weight. The spectral theorem supplies the equality of magnitudes. The functional measure must supply the sign and power.

6.2 Three possible realizations

6.2.1 First-order auxiliary-current formulation

A first-order quadratic action may be written schematically as

$$S^{(2)}[\eta, J, \lambda] = \frac{1}{2}\left\langle J, K^{-1}J\right\rangle + \mathrm{i}\left\langle \lambda, J - K d_0\eta\right\rangle, \tag{21}$$

where λ imposes the exact-current relation. Integrating different variables in different orders exposes Schur complements and Jacobians. This can prove that J adds no independent pole, but the exact determinant power depends on the chosen measure and reality conditions.

6.2.2 Constraint Jacobian

Changing variables from a fluctuation η to its image $D\eta$ produces a Jacobian proportional to a determinant of D on the non-zero subspace. If the physical measure divides by the volume of redundant directions, a determinant quotient can arise. The quotient must be derived rather than named.

6.2.3 BRST/BV complex

If the exact-current relation is generated by a gauge or cohomological redundancy, a nilpotent differential Q and Grassmann ghost fields may produce the required superdeterminant. This is the cleanest route to a genuine cancellation theorem, because the sign then follows from statistics and the measure is controlled by a known complex. It also creates a precise anomaly question: does the regulator preserve Q and the twisted return conditions?

6.3 No new poles

The safest form of the partnerless claim is structural:

Proposition 6.1 (No independent asymptotic current pole under exact-image realization)**.** *If the current variable is restricted to $J \in \operatorname{im} D$ and is introduced only as $J = D\eta$ or as an auxiliary field constrained to that image, then it does not define an independent LSZ sector. Any pole in a current correlator is inherited from the phase propagator and is not an additional particle pole.*

This proposition is much weaker than full S-matrix unitarity, but it directly supports the intended statement that hypersymmetry need not predict superpartners.

Open derivation target

Construct one explicit continuum or lattice functional measure for which the one-loop partition function factorizes into a superdeterminant of the phase–current complex. The derivation must state all fields, statistics, zero-mode volumes, seam boundary conditions, and regulator choices.

7 Auditable finite models

7.1 The unweighted four-cycle

Let C_4 be a cycle with four vertices and four oriented edges. With unit stiffness, the incidence matrix can be chosen as

$$B = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix}. \tag{22}$$

Then

$$H_0 = B^{\mathsf{T}} B, \qquad H_1 = B B^{\mathsf{T}},$$

and both spectra are

$$\{0, 2, 2, 4\}. \tag{23}$$

Thus

$$\det{}' H_0 = \det{}' H_1 = 16. \tag{24}$$

The phase zero mode is the constant vector. The current zero mode is the uniform circulation around the loop. The positive spectrum pairs exactly, while the zero modes retain distinct global meanings.

7.2 A weighted graph with two independent cycles

Consider four vertices, the four boundary edges of a square, and one diagonal. Let the edge stiffnesses be

$$K = \mathrm{diag}(1.0, 1.7, 0.6, 2.2, 1.3). \tag{25}$$

For $D = K^{1/2}B$, direct diagonalization gives

$$\mathrm{spec}(H_0) = \{0,\ 2.3061116264,\ 5.0121639615,\ 6.2817244121\}, \tag{26}$$

$$\mathrm{spec}(H_1) = \{0,\ 0,\ 2.3061116264,\ 5.0121639615,\ 6.2817244121\}. \tag{27}$$

The common primed determinant is

$$\det{}' H_0 = \det{}' H_1 = 72.608. \tag{28}$$

The connected graph has one phase zero mode and two independent cycle-current zero modes. Hence $\mathrm{ind}\, D = 1 - 2 = -1$.

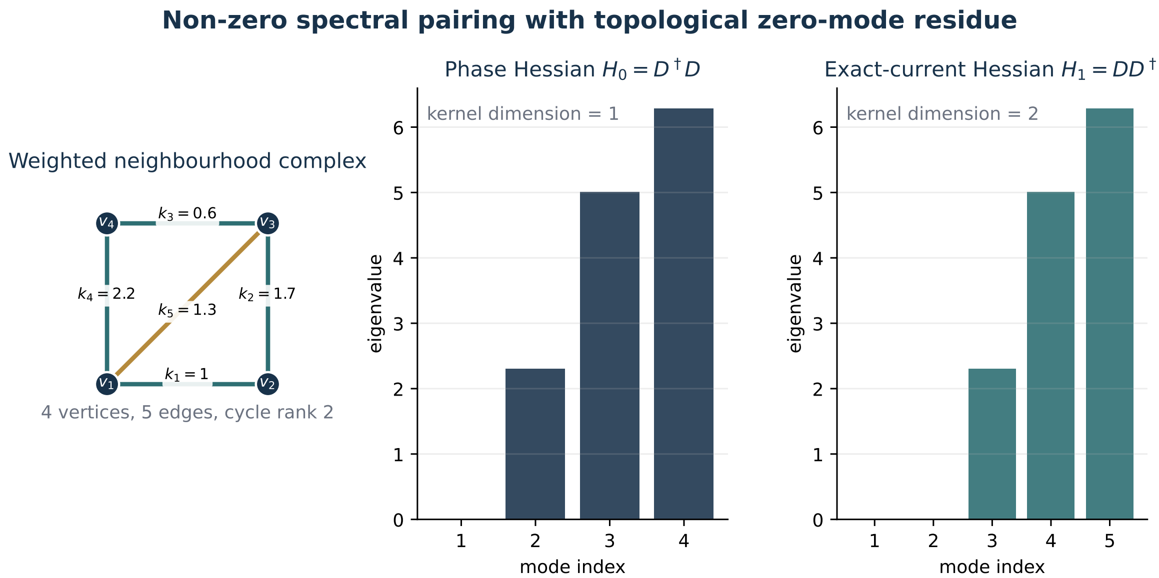

Figure 1: A weighted four-vertex, five-edge neighbourhood complex. The three positive eigenvalues of the phase Hessian and exact-current Hessian agree. The extra current zero mode records the second independent cycle.

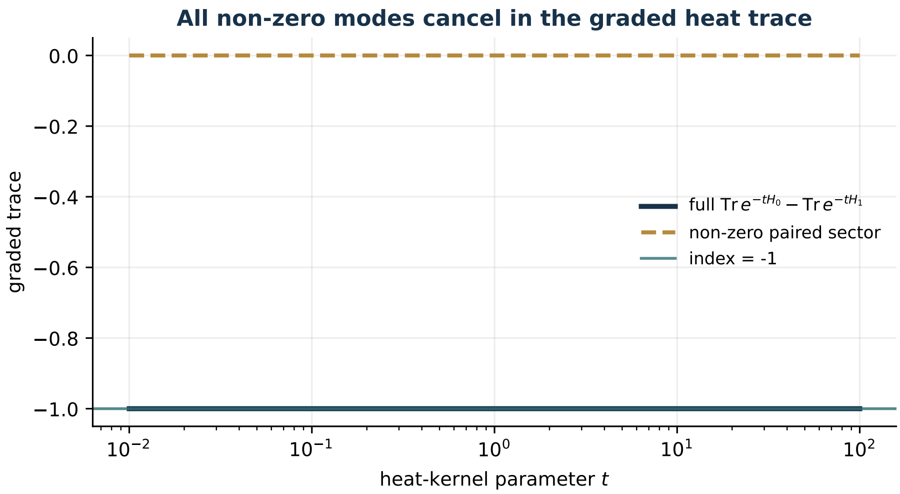

Figure 2: The non-zero paired heat trace vanishes for every t. The full graded trace remains equal to the index -1, determined entirely by the kernel dimensions.

7.3 Orientation flip as a numerical stress test

Multiplying any subset of the five incidence rows by -1 leaves H_0 unchanged and conjugates H_1 by a diagonal sign matrix. A reproducibility script should randomize all 2^5 orientation choices and verify identical spectra. This is a simple but important test: no physical conclusion may depend on the arbitrary arrow assigned to an edge.

7.4 A twisted Klein-bottle local system

The Klein bottle has a one-vertex cellular model with one-cells a, b and one two-cell attached by the word

$$r = aba^{-1}b, \qquad \pi_1(K_2) = \langle a, b \mid aba^{-1} = b^{-1} \rangle. \tag{29}$$

Let $\rho : \pi_1(K_2) \to U(1)$ be the one-dimensional local system

$$\rho(a) = 1, \qquad \rho(b) = -1. \tag{30}$$

The relation is satisfied because $(-1)^{-1} = -1$.

Using Fox derivatives,

$$\frac{\partial r}{\partial a} = 1 - b^{-1}, \qquad \frac{\partial r}{\partial b} = a + b^{-1}. \tag{31}$$

After evaluation in (30), the twisted cellular boundary maps are

$$0 \longrightarrow C_2 \simeq \mathbb{C} \xrightarrow{\partial_2} C_1 \simeq \mathbb{C}^2 \xrightarrow{\partial_1} C_0 \simeq \mathbb{C} \longrightarrow 0, \tag{32}$$

with

$$\partial_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \qquad \partial_1 = \begin{pmatrix} 0 & -2 \end{pmatrix}. \tag{33}$$

One immediately checks $\partial_1 \partial_2 = 0$. Moreover,

$$H_2(K_2; L_\rho) = 0, \qquad H_1(K_2; L_\rho) = 0, \qquad H_0(K_2; L_\rho) = 0. \tag{34}$$

The twisted complex is acyclic.

Established result

This calculation supplies a concrete non-mirror role for K_2. The return holonomy can be encoded by a local coefficient system, and a particular holonomy can lift all cohomological zero modes. Other representations may leave residues. Thus seam cancellation is a calculable property of the local system, not a universal consequence of saying "Klein bottle."

8 Euclideanity, Woit, and the emergence of M_4

8.1 The uncertainty-local regulator patch

The premetric Wistor complex is not assumed to be Euclidean. A Euclidean operator is introduced only after an operational Lorentzian chart has emerged and only on a patch where the quadratic fluctuation problem can be deformed to an elliptic one without crossing a zero mode.

Call a patch $U \subset M_4$ *uncertainty-local* for a mode scale k^{-1} when, schematically,

$$k^{-1}\|\nabla \ln K\| \ll 1, \qquad k^{-2}\|\mathcal{R}[g]\| \ll 1, \qquad [\gamma] = 0 \text{ for regulator loops } \gamma \subset U, \tag{35}$$

and the patch does not cross a caustic, defect, or non-trivial seam. On such a patch the Lorentzian quadratic operator may be contour-deformed to an elliptic H_E for heat-kernel or zeta regularization.

Explicit ΦBSU construction

Euclideanity is a local property of the regulated Hessian, not the ontology of the Separverse. Long loops, non-trivial holonomy, caustics, and source history must be restored through matching and cannot be assumed to live on one global Euclidean manifold.

8.2 Woit and Wistor: a useful difference in starting point

Woit's draft starts with a holomorphic Euclidean chiral theory and seeks a conjugation that reconstructs a Minkowski theory. The Wistor sequence begins with neighbourhood and phase-current compatibility, seeks a coherent causal M_4 chart, and only then Euclideanizes the local one-loop operator. The two viewpoints share a warning against naive inverse Wick rotation, but they address different layers.

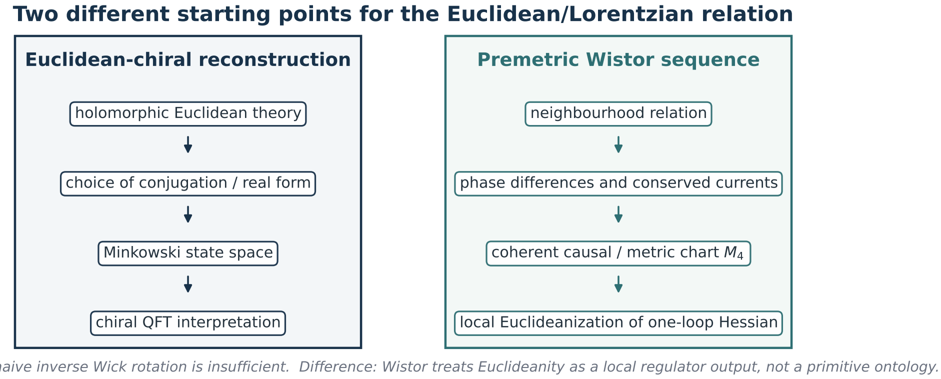

Figure 3: Woit's reconstruction question and the premetric Wistor question begin at different levels. The former is a problem inside a field-theoretic representation; the latter asks how the representation itself becomes available.

Woit's argument also demonstrates why anti-linear conjugation must not be confused with the cochain grading Γ. Osterwalder–Schrader conjugation helps define a physical Hermitian structure after Euclideanization. The one-loop phase–current pairing is already present before such a conjugation and is encoded by the linear map D.

8.3 Reflection positivity as a later consistency condition

If the emergent M_4 theory is constructed through an Euclidean functional integral, its Schwinger functions must satisfy a reflection-positivity condition strong enough to reconstruct a positive Hilbert space. K_2 return holonomy does not automatically imply that property. A plausible combined involution may involve Euclidean time reflection, complex conjugation, and the internal return local system, but positivity must be proved on the chosen algebra.

Open derivation target

Given an emergent M_4 background and a K_2-twisted phase-current measure, construct a reflection-positive Euclidean algebra or show why the physical theory should instead be defined directly in Lorentzian signature.

8.4 Chirality and CPT are emergent M_4 statements

Before an oriented Lorentzian metric and Clifford algebra exist, there is no γ_5, no Weyl decomposition, and no physical C, P, or T operation. The cochain relation $J_{xy} = -J_{yx}$ is not parity. It is the two-endpoint reading of one current.

Once a local Lorentzian field algebra is available, standard conditions may imply CPT. The appropriate programme-level statement is therefore conditional:

$$\text{locality} + \text{Lorentz covariance} + \text{unitarity} + \text{spectral assumptions on } M_4 \quad \Longrightarrow \quad \mathrm{CPT}_{M_4}. \tag{36}$$

CPT is a consistency property of the emergent operational theory, not a generator imposed on the primitive phase network.

9 Relation to supersymmetry and other cohomological mechanisms

The name hypersymmetry should be used with enough similarity to standard supersymmetry to be informative and enough difference to avoid category error.

Table 1: Comparison of ordinary supersymmetry and phase–current hypersymmetry.

Feature	Standard supersymmetry	ΦBSU one-loop hypersymmetry
Primitive grading	Boson/fermion or form-degree grading	Cochain degree: phase C^0 versus exact current $\operatorname{im} D \subset C^1$
Generator	Supercharge, often closing on translations	Neighbourhood differential $D = K^{1/2}d_0$
Paired objects	Distinct fields or states in a multiplet	A fluctuation and its generated exact current
Particle prediction	Often additional on-shell partners	None if current sector is auxiliary or constrained
Loop cancellation	Follows from statistics and supersymmetric measure	Conditional on a derived Jacobian or BRST/BV measure
Unpaired sector	BPS states / cohomology	Constant phase, cycles, seams, twisted cohomology
All-order protection	Possible in exact SUSY models	Not claimed; present mechanism is one-loop
Spacetime algebra	Super-Poincare or curved generalization	No fundamental spacetime algebra at premetric level

The closest established mathematical relatives are supersymmetric quantum mechanics, Hodge theory, and topological/cohomological field theory. Analytic torsion is especially suggestive because a ratio of determinants over a cochain complex naturally separates paired bulk modes from topological residue [12, 13]. That analogy should guide the next derivation, but it should not be used as a substitute for the actual ΦBSU measure.

Reading rule

The defensible phrase is *partnerless one-loop spectral stabilization*. The stronger phrase *one-loop cancellation* should be reserved for a version in which the functional measure has been derived.

10 The unification horizon

Hypersymmetry is not itself a complete unification theory. Its role is architectural: it offers a way for a phase-based theory to possess paired one-loop bulk fluctuations without duplicating the physical spectrum. The wider ΦBSU programme proposes that the same phase and separability grammar underlies gauge identity, buoyancy geometry, matter interiors, and large-scale support. These proposals have different evidential status and should remain separable.

10.1 Identity and curvature as the gauge bridge

Equation (1) supplies a clean programme-level bridge between a global phase and local gauge curvature. A successful derivation would need to show how A_{geom} emerges or is sourced from the stiffness and defect structure, why the operational gauge symmetry has the observed form, and how standard Ward identities arise in the M_4 chart.

Hypersymmetry contributes by organising the quadratic phase and current response. It does not derive the full nonlinear gauge action by itself.

10.2 The $3 \otimes \bar{3} = 1 \oplus 8$ colour node

A companion ΦBSU note interprets a 3^2 holonomy node through

$$\mathrm{End}(\mathbb{C}^3) = \mathbb{C}I \oplus \mathfrak{sl}(3, \mathbb{C}), \tag{37}$$

whose compact chart is the familiar singlet plus octet decomposition [5]. The representation identity is standard. The proposed novelty is to interpret the triplet as colour-tail terminals and the octet as traceless holonomy-control directions.

This proposal is compatible with the phase-current perspective: endomorphisms are transition operators on an internal phase space. But the derivation of quark dynamics, confinement, the Yang–Mills action, and the measured running coupling remains open. The algebraic match should be presented as a strong compatibility condition, not as a completed replacement for QCD.

10.3 Vacuum-structure numbers and matter amplitudes

The vacuum-structure note proposes numerical rules for the fine-structure constant and charged-lepton mass-amplitude ratios from a 2–3–5 information hierarchy [4]. These are sharp candidate predictions and therefore useful falsification targets. They are not consequences of the phase–current isospectrality theorem. Their relation to hypersymmetry is programmatic: both use the same phase, branch, and quadratic-readout grammar.

The correct synthesis is modular. A future failure of a numerical mass rule would not falsify the operator theorem. Conversely, the operator theorem would not rescue an incorrect numerical prediction.

10.4 Six-component ambient support and Part II

The quark–gluon companion note proposes a six-component ambient outerior from the rank of a primary $3T \oplus 3R$ base, while Part II explores sourcewise support, frozen hierarchy, galaxy benches, and drag-latched memory seeding [5, 3]. Again, these are not derived from hypersymmetry alone. The one-loop complex may help control fluctuations of the support field and separate bulk from topological memory, but the radial kernels and cosmological transfer functions require independent dynamical closure.

10.5 What a genuine unified derivation would look like

A successful unification would reduce the present modular correspondences to one variational and measure-theoretic structure. At minimum, a universal functional should determine

1. the neighbourhood stiffness $K[\alpha]$ and its nonlinear completion;
2. the emergence of a rank-four causal chart and its Lorentzian principal symbol;
3. the identity/curvature gauge split and the allowed gauge group representations;
4. the matter closure sectors and their asymptotic-state conditions;
5. the one-loop measure that realizes hypersymmetric determinant cancellation;
6. the twisted return topology and its anomaly phases;
7. and the cross-scale matching from particle interiors to ambient support.

That is the role assigned to κ_Φ. Until it is derived, κ_Φ should be treated as the name of the missing universal closure, not as an explanation by itself.

11 A claim ledger: results, constructions, conditions, and horizons

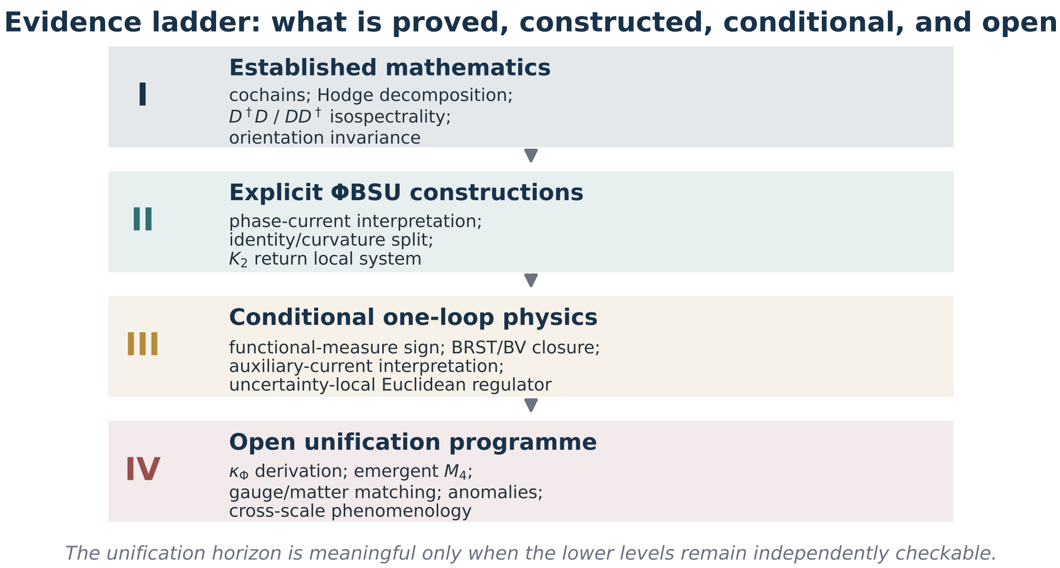

Figure 4: The evidence ladder used throughout this synthesis. Ambition at level IV is legitimate only if levels I–III remain explicit and independently testable.

Table 2: Claim ledger for the present hypersymmetry synthesis.

Claim	Status	What is currently established	Decisive next test
Edge-orientation independence	Established mathematics	$D \mapsto SD$ leaves $D^\dagger D$ invariant and conjugates $DD^\dagger$	Random orientation audit on finite complexes; continuum bundle analogue

Claim	Status	What is currently established	Decisive next test
Non-zero phase/current isospectrality	Established mathematics	Polar decomposition or eigenvector proof	Domain and continuum-limit control for the chosen κ_Φ Hessian
Heat-kernel cancellation in graded trace	Established mathematics	Non-zero paired traces agree; supertrace equals index	Correct handling of continuous spectrum and boundaries
K_2 as twisted return topology	Explicit construction	Cellular local systems can be written and their cohomology computed	Select physically motivated representation and seam conditions
No new current particle	Conditional construction	True if current is exact image or auxiliary constrained field	Integrate the first-order action and inspect the full propagator matrix
One-loop determinant cancellation	Conditional physics	Requires opposite determinant weights beyond isospectrality	Derive Jacobian or BRST/BV superdeterminant
Regulator compatibility	Conditional physics	Can require commutation with cochain and return operators	Exhibit a concrete regulator preserving all Ward and seam identities
Anomaly cancellation	Open	No universal cancellation follows from orientation reversal alone	Compute determinant line / η phase for a complete emergent fermion content
Emergent Lorentzian M_4	Open programme	Operational charts exist in current papers; derivation not complete	Show rank-four hyperbolic principal symbol from κ_Φ and clock sector
CPT emergence	Open/conditional	Expected only after local Lorentzian field algebra is established	Verify locality, positivity/unitarity, spectral condition, and spin-statistics
Gauge and matter unification	Open programme	Identity/curvature split and representation compatibilities are explicit	Derive nonlinear action, representations, couplings, and RG behaviour
Cross-scale phenomenology	Mixed numerical programme	Part II and companion notes supply benchmarks and predictions	Blind data tests, uncertainty propagation, and independent reproduction

12 Falsification and the next research cycle

A research programme becomes sharper when it specifies how it can fail. The following tests are ordered from the mathematical core outward.

12.1 Gate 1: finite-complex reproducibility

Publish a minimal code package that constructs incidence matrices, positive stiffness matrices, H_0, H_1, primed determinants, and heat supertraces. Include orientation randomization, boundary cases, disconnected graphs, and twisted local systems. Any failure of non-zero pairing under the stated hypotheses is a mathematical error and must stop the programme.

12.2 Gate 2: physical measure

Write an explicit quadratic path integral with all fields and statistics. Derive the determinant quotient. If the measure gives a sum rather than a difference, hypersymmetry remains a spectral pairing but not a vacuum-energy cancellation mechanism.

12.3 Gate 3: continuum convergence

Choose a family of refining complexes and prove or numerically demonstrate spectral convergence to a continuum operator. Verify that the zero-mode sector converges to the intended cohomology rather than producing uncontrolled lattice artefacts.

12.4 Gate 4: emergent causal chart

Derive the operational M_4 metric or principal symbol from κ_Φ without presupposing the same metric in the definition of the primitive density. A self-consistent implicit equation is acceptable, but it must have existence, signature, and stability conditions.

12.5 Gate 5: gauge/BRST and anomaly consistency

Show that the emergent gauge action and the hypersymmetric measure share a regulator that preserves the necessary Ward or BRST identities. Compute local anomaly polynomials and global determinant-line phases for the actual field content. Topological slogans cannot replace this calculation.

12.6 Gate 6: independent phenomenology

Separate predictions fixed before data inspection from exploratory fits. Supply machine-readable inputs, uncertainty budgets, competing baselines, and negative controls. The fine-structure and lepton ratios, quark shape rules, galaxy support hierarchy, and early-seeding claims should be evaluated as distinct modules, not pooled into one impression of success.

The central derivation problem

Find a universal κ_Φ and functional measure whose second variation yields the phase–current complex, whose coarse principal symbol yields an operational Lorentzian M_4, and whose gauge-fixed one-loop measure produces the required superdeterminant without adding physical poles. This single derivation would convert the present synthesis from a coherent programme map into a genuine unification mechanism.

13 Conclusion

The strongest current form of ΦBSU hypersymmetry is both more modest and more durable than the original pseudo-mirror formulation. It is the statement that a phase fluctuation and its exact neighbour current are the two sides of one quadratic update map. The operators $D^\dagger D$ and $DD^\dagger$ share every non-zero spectral mode. Their difference is topological: constants, cycles, seams, and twisted cohomology.

This result stands squarely on established mathematics. Its novelty is interpretive and architectural. It allows one-loop pairing without postulating a superpartner spectrum, a fundamental mirror, or primitive chirality. It also fits naturally with a background-independent research direction in which signs and orientations are auxiliary until an operational M_4 chart emerges.

The central limitation is equally clear. Isospectrality is not yet a physical determinant cancellation. The functional measure must provide the opposite weight. Nor does the pairing derive the metric, gauge group, matter spectrum, or observed constants. Those belong to the larger κ_Φ problem.

A useful unification essay should therefore end neither with triumph nor with retreat. The correct conclusion is a disciplined opening:

proved phase–current pairing $\longrightarrow$ extbfderived one-loop measure
$\longrightarrow$ extbfemergent M_4 and gauge chart $\longrightarrow$ extbftestable unification

The first arrow is now concrete enough to audit. The remaining arrows are ambitious, but they have been turned into calculations rather than protected aspirations. That is an appropriate form for a curiosity-driven programme standing on giants' shoulders while still reaching beyond the current paradigm.

A Notation and status vocabulary

Symbol / term	Meaning
$\mathcal{C}$	Premetric neighbourhood or cell complex used to represent local separability relations
$C^p(\mathcal{C})$	p-cochains on the complex
d_0	Neighbourhood coboundary mapping vertex phases to edge differences
K	Positive linearized phase-stiffness operator on one-cochains
$D = K^{1/2}d_0$	Weighted phase-to-current amplitude map
$H_0 = D^\dagger D$	Phase-fluctuation Hessian
$H_1 = DD^\dagger$	Exact-current Hessian
$\mathcal{D}_\mathcal{N}$	Cochain Dirac operator on $C^0 \oplus C^1$
K_2	Klein-bottle return topology or twisted seam bookkeeping; not a physical mirror world
Wistor	Premetric phase-current engine; not Penrose twistor space
M_4	Emergent operational Lorentzian meter chart
uncertainty-local	Patch on which the one-loop Hessian can be locally Euclideanized without crossing global defects or zero modes
Established result	Follows from stated mathematical hypotheses and can be independently checked
Explicit construction	A fully written ΦBSU model element, but not forced by the theorem
Conditional statement	Follows if a separately stated measure, matching, or emergence condition is satisfied
Open target	A missing derivation or empirical test

B Matrix data for the weighted example

For the graph in figure 1, the incidence matrix is

$$B = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{pmatrix}, \qquad K = \mathrm{diag}(1, 1.7, 0.6, 2.2, 1.3). \tag{38}$$

The exact numerical positive eigenvalues are

$$2.3061116263805, \qquad 5.0121639615412, \qquad 6.2817244120783. \tag{39}$$

Their product is 72.608. The cycle rank is $E - V + 1 = 2$, matching $\dim\ker H_1 = 2$ for a connected graph. The constant phase gives $\dim\ker H_0 = 1$.

C Minimal reproducibility pseudocode

```
input: oriented incidence matrix B (E x V)
```

```
       positive edge stiffness K (E x E)

D  = sqrtm(K) @ B
H0 = transpose(D) @ D
H1 = D @ transpose(D)

lambda0 = eigvalsh(H0)
lambda1 = eigvalsh(H1)

assert positive_spectrum(lambda0) == positive_spectrum(lambda1)

for every diagonal sign matrix S on edges:
    Dp  = S @ D
    H0p = transpose(Dp) @ Dp
    H1p = Dp @ transpose(Dp)
    assert H0p == H0
    assert spectrum(H1p) == spectrum(H1)

for t on a logarithmic grid:
    graded_trace(t) = trace(exp(-t H0)) - trace(exp(-t H1))
assert graded_trace(t) == dim(kernel(H0)) - dim(kernel(H1))
```

For the twisted Klein example, implement the boundary matrices in equation (33) and verify

```
partial1 @ partial2 == 0
rank(partial2) == 1
rank(partial1) == 1
H0_dim == H1_dim == H2_dim == 0
```

D A suggested structure for a future referee paper

A later narrow submission can be extracted from this essay without carrying the full unification discussion. Its core could be:

1. premetric neighbourhood complex and orientation invariance;
2. phase–current isospectrality theorem;
3. weighted finite examples and twisted K_2 local system;
4. explicit first-order or BRST/BV measure;
5. uncertainty-local continuum matching;
6. limitations and falsification criteria.

The broader gauge, matter, and cosmological programme would then appear only in a final outlook section.

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