ΦBSU **RESEARCH ESSAY - REVISED SYNTHESIS v1.1**

One-Loop Hypersymmetry

in the ΦBSU Unification Programme

Reciprocal phase closure, premetric phase-current pairing, and the non-physical status of holonomy seams

Revision focus. A completed bidirectional M_4 comparator has total phase exactly unity. A measurable phase displacement is therefore not an uncompensated defect: it is balanced by material-state change, momentum transfer, and/or bosonic zero-fibre support energy. The distribution of that compensation may remain fuzzy, but the completed comparator closes. In this revision, K_2 seams are consequently treated as transition-function and return-topology bookkeeping rather than as physical mirror sectors or independent anomaly reservoirs.

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Revised example synthesis based on the current Part I, Part II, original hypersymmetry draft, vacuum-structure, quark-gluon, and Woit discussion materials.

Tiivistelmä

Tässä revisiossa ΦBSU-hypersymmetria esitetään yhden silmukan vaihe–virtaparituksena ilman fundamentaalista peiliä, kätisyyttä tai superpartnerispektriä. Premetrisessä naapuruuskompleksissa vaihefluktuaatio $\eta \in C^0$ tuottaa tarkan naapuruusvirran $D\eta \in C^1$, ja operaattorit

$$H_0 = D^\dagger D, \qquad H_1 = DD^\dagger$$

ovat nollasta poikkeavalla sektorilla isospektraalisia. Tämä on tutkielman varsinainen tarkistuskelpoinen hypersymmetriatulos.

Revision keskeinen kirkastus on M_4-rakenneresonanssin kaksisuuntainen sulkeuma. Valmiille komparaattorille vaaditaan

$$\mathscr{H}_{M_4}(\Gamma;\lambda) = U_{\rightarrow}U_{\leftarrow}U_{\mathrm{state}}U_{\mathrm{mom}}U_{0\mathrm{f}} = \mathbf{1}$$

jokaiselle sallitulle mikrohistorialle λ. Täsmällinen kokonaisvaihe ei merkitse vaihesiirtojen puuttumista. Se merkitsee, ettei mikään vaihesiirto jää ilman vastakirjausta aineen olomuotoon, liikemäärään tai bosoniseen nollakuituenergiaan. Sumeus voi säilyä siinä, miten kompensaatio jakautuu kanavien ja $\sqrt{2}$-erillistyvien tukitasojen kesken.

Tämä palauttaa holonomiasaumat yksinkertaisempaan asemaan. Sauma on paikallisten vaihe-esitysten siirtymäfunktio, laskennallinen leikkaus tai twisted-paikallissysteemin palautusehto. Se ei ole itsenäinen energiasektori eikä piilotettu fysikaalinen peililehti. Suljetun komparaattorin kokonaisvaihe voi olla 1 myös ei-triviaalissa winding-luokassa, joten topologista tietoa ei pyyhitä pois.

Tutkielma erottaa kolme kvanttikysymystä. Spektriparitus koskee kvadraattisten operaattorien nollasta poikkeavia ominaisarvoja. Funktionaalimitan täytyy erikseen tuottaa vastakkainen determinanttipaino, jos one-loop-moduulin halutaan kumoutuvan. Kokonaisvaiheen 1-sulkeuma puolestaan koskee valmiin fysikaalisen komparaattorin vaihetta eikä yksin korvaa emergentin kiraalisen M_4-teorian tavallista gauge-anomaliatarkastusta.

Mukana säilytetään kaksi äärellistä tarkistusmallia. Painotetussa neljän solmun ja viiden särmän verkossa H_0:n ja H_1:n kolme positiivista ominaisarvoa yhtyvät, kun taas virtasektoriin jää kaksi syklinollamoodia. Klein-pullon lokaalissysteemissä $\rho(a) = 1$, $\rho(b) = -1$ twisted-kompleksi on asyklinen ja sen standardimetrinen torsio on $T = 1$. Tämä tulos sopii revision perusviestiin: valittu epäfysikaalinen palautussauma voi poistaa kirjanpidolliset nollamoodit jättämättä uutta fysikaalista sektoria.

Euklidisointi rajataan emergentin M_4-kartaston epävarmuus-paikalliseen one-loop-patchiin. Peter Woitin euklidis-holomorfinen rekonstruktio käsitellään myöhemmän tason varoituksena, ei Wistor-moottorin ontologisena lähtökohtana. Kätisyys, C, P, T ja CPT ovat vasta paikallisen Lorentzisen mittarikenttäalgebran käsitteitä.

Revision principle

Ydinlause: vaihevirhe kumoutuu, mutta kompensoiva liike säilyy. Hypersymmetria parittaa one-loop-bulk-moodit; M_4-rakenneresonanssi sulkee valmiin tapahtuman kokonaisvaiheen; sumea tukiambientti voi kantaa kompensaation jakautumaa ilman uusia LSZ-poleja.

Abstract

This revision presents ΦBSU hypersymmetry as a one-loop phase-current pairing without a fundamental mirror, handedness, or superpartner spectrum. On a premetric neighbourhood complex, a phase fluctuation $\eta \in C^0$ generates an exact current $D\eta \in C^1$, and the quadratic operators $H_0 = D^\dagger D$ and $H_1 = DD^\dagger$ have identical non-zero spectra. That is the principal auditable result.

The central simplification is reciprocal M_4 phase closure. A completed bidirectional comparator obeys

$$\mathscr{H}_{M_4}(\Gamma;\lambda) = U_{\rightarrow}U_{\leftarrow}U_{\mathrm{state}}U_{\mathrm{mom}}U_{0\mathrm{f}} = \mathbf{1}$$

for every admissible microhistory λ. Exact total phase does not eliminate measurable phase displacements. It means that no displacement remains uncompensated: it is transferred into material-state change, momentum, and/or bosonic zero-fibre support energy. Fuzziness may remain in the channel and scale distribution of the compensation, including a proposed $\sqrt{2}$-separated support ambient, while the completed comparator closes exactly.

This clarification restores holonomy seams to a non-physical role. A seam is a transition function between local phase representatives, a bookkeeping cut, or a twisted return condition. It is not an independent energy-bearing mirror sheet. Non-trivial winding may survive even when the total group element is unity.

The essay separates spectral pairing, determinant modulus, and total phase. Isospectrality does not supply a minus sign in the functional measure. An explicit calculation of the naive commuting first-order measure reproduces the original phase determinant and gives no cancellation. Reciprocal closure fixes the phase of a completed physical comparator, but it does not replace the ordinary representation-theoretic and determinant-line anomaly audit of an emergent chiral M_4 theory.

Two finite models remain fully auditable. A weighted graph exhibits identical positive spectra for H_0 and H_1 with topological current zero modes. A twisted Klein-bottle local system with $\rho(a) = 1$ and $\rho(b) = -1$ is acyclic and has unit torsion in the standard metric. This is interpreted as a bookkeeping baseline: the seam can remove unphysical return zero modes without leaving a new physical sector.

Euclideanization is restricted to an uncertainty-local one-loop regulator patch after an operational Lorentzian M_4 chart has emerged. Woit's Euclidean-holomorphic reconstruction is treated as a useful later-stage warning rather than the ontological origin of the Wistor engine. Chirality and CPT are emergent properties of the local Lorentzian field algebra.

Keywords: hypersymmetry; reciprocal phase closure; phase-current pairing; holonomy seams; background independence; Wistor; one-loop determinant; fuzzy support ambient; emergent M_4.

Contents

1 Revision thesis: simplify the physical picture before extending it

The earlier synthesis correctly separated established mathematics, explicit constructions, conditional physics, and open unification targets. Its remaining weakness was not excessive mathematical care but a tendency to let several distinct problems accumulate around the word *holonomy*: one-loop determinant pairing, anomaly phases, non-orientable return topology, fuzzy ambient support, and Lorentzian reconstruction began to appear as if they required a single elaborate mechanism.

The present revision adopts a stronger organising fact supplied by the ΦBSU model itself:

$$\boxed{\text{a completed bidirectional } M_4 \text{ comparator has total phase } \mathbf{1}.} \tag{1}$$

A local or partial phase displacement may be non-zero and observable. What is forbidden is an *uncompensated* phase displacement in the completed event. The missing balance is carried as motion or support: a change of material state, momentum exchange, curvature-field energy, or bosonic zero-fibre energy. This replaces the image of two physical sheets cancelling one another with a single event whose different bookkeeping channels close together.

The revision therefore uses three distinct statements:

1. **Kinematic one-loop pairing:** $D^\dagger D$ and $DD^\dagger$ have the same non-zero spectrum.
2. **Dynamic reciprocal closure:** outward signal, return signal, material response, momentum response, and zero-fibre response multiply to the identity.
3. **Quantum consistency of emergent M_4:** the functional measure, Ward identities, anomalies, positivity, and CPT must be checked in the ordinary field-theoretic way appropriate to the derived M_4 sector.

They may ultimately descend from one universal stiffness functional κ_Φ, but they should not be used as substitutes for one another before that derivation exists.

Revision principle

The non-physical seam is not asked to carry energy, cancel a chiral anomaly, or provide a hidden partner spectrum. Its minimal job is to relate local phase representatives and encode the return rule of a completed comparator. This is enough to preserve holonomy bookkeeping while keeping the physical dynamics local.

2 The inherited paradigm: the shoulders of the proposal

2.1 Gauge connections, curvature, and global comparison

The operational field dictionary of ΦBSU separates a global identity connection from local curvature,

$$A = A_{\text{geom}} + A_{\text{id}}, \qquad A_{\text{id}} = -\mathrm{i}\Phi^{-1}\mathrm{d}\Phi = \mathrm{d}\alpha, \qquad F_{\text{geom}} = \mathrm{d}A_{\text{geom}}. \tag{2}$$

On a smooth simply connected patch, $\mathrm{d}A_{\text{id}} = 0$. A relative phase can nevertheless survive a closed comparison when the phase is multi-valued or the domain contains non-trivial topology. The standard lesson is that local curvature and global phase comparison are different data. The ΦBSU extension is to interpret A_{id} as identity bookkeeping and F_{geom} as the locally energy- and momentum-carrying channel [2].

The reciprocal-closure revision adds a crucial qualification. The phase around a *partial* comparator may be non-trivial. The completed physical event includes the source, detector, momentum, and support response, and its total group element closes to the identity. This does not erase Aharonov–Bohm-type relative phases; it says that the complete apparatus-plus-field history remains a consistent closed event.

2.2 One-loop effective actions and spectral complexes

For a quadratic bosonic operator Δ the formal Euclidean one-loop contribution is

$$\Gamma^{(1)}_{\mathrm{bos}} = \frac{1}{2}\,\mathrm{Tr}\ln\Delta, \tag{3}$$

whereas Grassmann integration gives an opposite determinant power. Heat kernels and zeta determinants organize the ultraviolet spectrum,

$$\ln\det{}'\Delta = -\int_0^\infty \frac{\mathrm{d}t}{t}\,\mathrm{Tr}'\mathrm{e}^{-t\Delta}, \tag{4}$$

with zero modes treated separately. The distinction between non-zero spectral pairing, zero-mode residue, determinant modulus, and determinant phase is therefore basic rather than optional.

2.3 Supersymmetry, Hodge theory, and the relevant inherited identity

Supersymmetric quantum mechanics and Hodge theory use the operator pair

$$D^\dagger D \quad \longleftrightarrow \quad DD^\dagger. \tag{5}$$

The positive spectra coincide, while kernels record topology. This mathematics is classical [8, 9]. The proposed novelty is not the singular-value theorem. It is the interpretation of the two spaces as a phase configuration and its exact neighbour-current response rather than as a physical boson and a physical fermion.

2.4 Wick rotation, reconstruction, and the limited role of twistor geometry

Woit's 2026 preliminary note stresses that ordinary inverse Wick rotation is not a simple analytic continuation, especially for chiral fields, and proposes a conjugation-based route from a Euclidean holomorphic description to a Minkowski state space [7]. This is a useful warning about the later field-theoretic reconstruction problem. It does not supply the premetric origin of neighbourhood, phase transport, or the M_4 comparator. The present Wistor construction starts before Euclideanity, chirality, and a Penrose twistor ambient are available.

3 The premetric Wistor engine

3.1 Neighbourhood before metric

Let C be a finite or locally finite cell complex used as an auxiliary representation of an unoriented neighbourhood structure,

$$C^0(C) \xrightarrow{d_0} C^1(C) \xrightarrow{d_1} C^2(C), \qquad d_1 d_0 = 0. \tag{6}$$

A phase is $\alpha \in C^0$. The edge difference is $d_0\alpha$, and a positive stiffness operator K defines the current

$$J = K d_0 \alpha. \tag{7}$$

The edge arrows are computational choices. If S is a diagonal sign matrix reversing any subset of edge orientations, then

$$d_0 \mapsto S d_0, \qquad J \mapsto SJ, \tag{8}$$

while the physical divergence and spectra below remain unchanged.

Definition 3.1 (Wistor)**.** *In this essay,* Wistor *denotes the premetric neighbourhood phase-current engine* (C, d, K, α)*. It is not Penrose projective twistor space. Ordinary spinor or twistor coordinates may become efficient charts only after a coherent causal geometry has emerged.*

3.2 Local mediation and globally extended invariants

For a source-free current, $d_0^\dagger J = 0$. Summing over a region cancels every internal edge with opposite endpoint signs, leaving only the boundary flux. The dynamics is neighbour-mediated, but the integrated invariant can depend on an extended region or closed loop. This is the precise sense in which the model combines locality of influence with global holonomy information.

For a closed edge cycle γ,

$$\mathrm{Hol}_\gamma(\alpha) = \exp\left(\mathrm{i} \sum_{e\in\gamma} (d_0\alpha)_e\right). \tag{9}$$

No instantaneous influence across the loop is implied. The quantity is operational only when locally transported histories return to a common comparator.

3.3 Premetric density and buoyancy as proposed coarse variables

A metric-free local stiffness norm can be represented schematically by

$$\rho_x^2 = \langle d_0\alpha, K d_0\alpha\rangle_{\mathrm{star}(x)}, \qquad a_e = -d_0 \ln \rho(e). \tag{10}$$

In an operational continuum chart these become the familiar programme variables

$$\rho = \|\nabla\alpha\|, \qquad a_\mu = -\partial_\mu \ln \rho. \tag{11}$$

The direction of derivation must be kept explicit: equation (10) is proposed as a premetric antecedent; the continuum metric norm is a later chart. A full proof requires κ_Φ to generate a rank-four hyperbolic principal symbol rather than presupposing one.

4 Reciprocal M_4 phase closure

4.1 The completed comparator

Let Γ_{xy} denote a completed bidirectional measurement history from a source or clock at x to a responder at y and back to a common readout. Let λ label the microscopic support history. The proposed structural resonance is

$$\boxed{\mathscr{H}_{M_4}(\Gamma_{xy};\lambda) = U_{x\to y}\, U_{y\to x}\, U_{\mathrm{state}}\, U_{\mathrm{mom}}\, U_{0\mathrm{f}} = \mathbf{1}.} \tag{12}$$

The factors need not commute in a future non-Abelian completion; their order is part of the comparator. In an Abelian phase chart,

$$\Delta\theta_\to + \Delta\theta_\leftarrow + \Delta\theta_{\mathrm{state}} + \Delta\theta_{\mathrm{mom}} + \Delta\theta_{0\mathrm{f}} = 2\pi N, \qquad N \in \mathbb{Z}. \tag{13}$$

Thus the total group element is unity even when the winding label N is non-zero.

Explicit ΦBSU construction

Equation (12) is a model postulate of the present revision, not a consequence of the phase-current isospectrality theorem. It formalizes the stated physical rule that every phase displacement carries a compensating movement or support response.

4.2 Phase displacement as transduction, not erasure

A phase shift is read as transfer between channels,

$$\Delta\theta_{\text{signal}} \longleftrightarrow \left(\Delta E_{\text{state}}, \Delta p^{\mu}, \Delta E_{0\text{f}}\right). \tag{14}$$

For variations that do not change the winding class,

$$\delta\theta_{\rightarrow} + \delta\theta_{\leftarrow} + \delta\theta_{\text{state}} + \delta\theta_{\text{mom}} + \delta\theta_{0\text{f}} = 0. \tag{15}$$

At the coarse stress-energy level the intended local counterpart is

$$\nabla_{\mu}\left(T^{\mu\nu}_{\text{signal}} + T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{geom}} + T^{\mu\nu}_{0\text{f}}\right) = 0. \tag{16}$$

Equation (16) is an open continuum closure target. Its purpose is to make clear that "phase compensation" must become ordinary local energy-momentum bookkeeping in the emergent chart.

4.3 Strong closure and the survival of fuzziness

The strong statement is

$$\mathscr{H}_{M_4}(\Gamma;\lambda) = \mathbf{1} \quad \text{for every admissible } \lambda, \tag{17}$$

not merely

$$\left\langle \mathscr{H}_{M_4}(\Gamma;\lambda)\right\rangle_{\lambda} = \mathbf{1}. \tag{18}$$

The latter would permit uncompensated individual histories whose phases cancel only statistically. The former allows a broad distribution $w(\lambda)$ of support histories while requiring each completed event to close.

Corollary 4.1 (Exact closure does not sharpen the ambient)**.** *If equation (17) holds pointwise in support-history space, the probability or amplitude distribution $w(\lambda)$ may remain broad. Exact total phase constrains the product of channel responses, not the microscopic location, scale, or partition of the compensation.*

4.4 Operational M_4 standards

A clock, ruler, frequency standard, or momentum calibration is not assigned to one directed signal segment. It is associated with an equivalence class of completed reciprocal comparators. Schematically,

$$\mathcal{M}_4^{\text{op}} = \left\{\Gamma_{xy} \,\middle|\, \mathscr{H}_{M_4}(\Gamma_{xy};\lambda) = \mathbf{1} \text{ for admissible } \lambda\right\} /\!\sim . \tag{19}$$

Equation (19) is not yet a construction of a differentiable four-manifold. It states the operational invariant that a future coarse-graining must preserve. The open geometric task is to show that the stable comparator family has four effective coordinates and a Lorentzian hyperbolic principal symbol.

4.5 Holonomy seams as transition functions

Cover a comparator domain by local charts U_i with phase representatives α_i. On overlaps,

$$\alpha_i - \alpha_j = \chi_{ij}, \qquad g_{ij} = \mathrm{e}^{\mathrm{i}\chi_{ij}}. \tag{20}$$

A drawn seam is a choice of where the representative changes. It is not by itself a physical surface. For a closed comparator,

$$\left(\prod_{(ij)\in\Gamma} g_{ij}\right) U_{\mathrm{dyn}}(\Gamma) = \mathbf{1}, \tag{21}$$

where U_{dyn} contains the local signal and compensating response. Different seam placements change the representative factors but not the completed product.

Established / auditable result

A non-physical holonomy seam may be moved like a branch cut without moving energy or matter. It becomes physically active only where the local curvature, source, detector, or defect sector supplies a genuine stress-energy or interaction term. The seam therefore carries bookkeeping, not an automatic hidden dynamics.

5 Hypersymmetry redefined: the one-loop phase-current theorem

5.1 Quadratic stiffness map

Let α_0 be a background phase configuration and η a small fluctuation. Assume the stiffness functional admits a stable quadratic expansion

$$S_\Phi[\alpha_0 + \eta] = S_\Phi[\alpha_0] + \frac{1}{2}\langle d_0\eta, K_{\alpha_0} d_0\eta\rangle + O(\eta^3), \tag{22}$$

where K_{α_0} is positive and self-adjoint on the local stable sector. Define

$$D = K_{\alpha_0}^{1/2} d_0, \qquad H_0 = D^\dagger D, \qquad H_1 = DD^\dagger. \tag{23}$$

Theorem 5.1 (Phase-current non-zero spectral pairing)**.** *Let $D : C^0 \to C^1$ be a closed densely defined operator with adjoint $D^\dagger$. Then $H_0 = D^\dagger D$ and $H_1 = DD^\dagger$ have identical non-zero spectra with equal multiplicities. For suitable functions f,*

$$\mathrm{Tr}'\, f(H_0) = \mathrm{Tr}'\, f(H_1), \qquad \det{}' H_0 = \det{}' H_1. \tag{24}$$

Proof. If $H_0 u = \lambda u$ with $\lambda > 0$, then $Du \neq 0$ and

$$H_1(Du) = DD^\dagger Du = D(H_0 u) = \lambda Du.$$

The inverse map on the λ eigenspaces is $v \mapsto \lambda^{-1/2} D^\dagger v$. Equivalently, the positive eigenvalues are the squared non-zero singular values of D. □

5.2 Cochain Dirac operator and topological residue

Define

$$\mathcal{D}_N = \begin{pmatrix} 0 & D^\dagger \\ D & 0 \end{pmatrix}, \qquad \Gamma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{25}$$

Then

$$\mathcal{D}_{\mathcal{N}}^2 = \begin{pmatrix} H_0 & 0 \\ 0 & H_1 \end{pmatrix}, \qquad \{\Gamma, \mathcal{D}_{\mathcal{N}}\} = 0. \tag{26}$$

The grading is the linear distinction between zero- and one-cochains. It is not spatial parity, chirality, charge conjugation, time reversal, or a physical mirror operation.

On the non-zero sector,

$$\operatorname{Str}' \mathrm{e}^{-t\mathcal{D}_{\mathcal{N}}^2} = 0. \tag{27}$$

Including kernels,

$$\operatorname{Str} \mathrm{e}^{-t\mathcal{D}_{\mathcal{N}}^2} = \dim\ker D - \dim\ker D^\dagger = \operatorname{ind} D. \tag{28}$$

The residue records constants, cycles, boundary conditions, and twisted cohomology.

Established / auditable result

The theorem establishes partnerless *spectral pairing*. It does not yet establish cancellation in a physical functional integral. The result is nevertheless robust: it requires no mirror sheet, no opposite handedness, and no additional asymptotic pole.

5.3 Orientation invariance

For an edge-sign matrix S with $S^\dagger S = 1$, set $D' = SD$. Then

$$H_0' = D'^\dagger D' = H_0, \qquad H_1' = D' D'^\dagger = S H_1 S^\dagger. \tag{29}$$

Thus all spectral statements are independent of the arbitrary arrow choices used to write the complex. This realizes the absence of a fundamental sign or rotation sense at the premetric level.

6 The physical measure: what cancels and what does not

6.1 Isospectrality does not create a determinant minus sign

The equality

$$\det{}' H_0 = \det{}' H_1 \tag{30}$$

does not determine whether the effective action contains a sum or a difference. Two commuting Gaussian fields add their $\frac{1}{2}\ln\det$ contributions. An opposite sign requires a Grassmann field, a Faddeev–Popov determinant, a constraint Jacobian, or an explicitly derived superdeterminant.

6.2 The naive commuting first-order measure

Consider the quadratic first-order action

$$S^{(2)}[\eta, J, \lambda] = \frac{1}{2}\langle J, K^{-1} J\rangle + \mathrm{i}\langle \lambda, J - K d_0 \eta\rangle. \tag{31}$$

With commuting variables and flat measures,

$$Z_{\text{1st}} = \int \mathcal{D}\eta\, \mathcal{D}J\, \mathcal{D}\lambda\, \mathrm{e}^{-S^{(2)}}. \tag{32}$$

Integrating λ gives $\delta(J - K d_0 \eta)$, and integrating J then gives

$$Z_{\text{1st}} \propto \int \mathcal{D}\eta\, \exp\left[-\frac{1}{2}\langle d_0\eta, K d_0 \eta\rangle\right] \propto \operatorname{Vol}(\ker D)\, [\det{}' H_0]^{-1/2}. \tag{33}$$

Established / auditable result

The naive first-order rewriting is equivalent to the original phase Gaussian. It introduces no new physical pole, but it also produces no compensating H_1 determinant. This closes one ambiguity in the earlier essay: equation (31) alone is not the hypersymmetric cancellation mechanism.

6.3 Exact-current coordinates and the risk of a trivial resolution

On the orthogonal complement of $\ker D$, the change of variables $J = D\eta$ has Jacobian

$$\mathcal{D}J_{\text{exact}} = (\det{}' H_0)^{1/2}\mathcal{D}\eta_\perp. \tag{34}$$

Using a canonically normalized current measure can therefore remove the phase determinant. But this may be no more than a coordinate resolution of the identity. A BRST doublet can do the same thing: contractible modes disappear from cohomology because they were never physical.

Definition 6.1 (Non-triviality criterion). *A measure realization of hypersymmetry is physically non-trivial only if the contractible exact bulk sector cancels while a regulator-independent residue survives and couples to at least one operational observable. Candidate residues include a harmonic mode, a seam holonomy, a torsion modulus, a finite form factor, or a zero-fibre support response.*

The desired structure is

$$Z^{(1)} = Z_{\text{exact}}\, Z_{\text{residue}}, \qquad Z_{\text{exact}} = 1, \qquad Z_{\text{residue}} \not\equiv 1 \tag{35}$$

for at least some admissible backgrounds. The possibility $Z_{\text{residue}} = 1$ is not an inconsistency; it means the chosen finite model is a pure bookkeeping baseline.

6.4 Determinant modulus versus reciprocal phase closure

Write a one-loop contribution schematically as

$$Z^{(1)} = |Z^{(1)}|e^{i\Theta}. \tag{36}$$

The phase-current measure problem concerns $|Z^{(1)}|$, including ultraviolet determinants and Jacobians. Reciprocal M_4 closure concerns the total physical phase of a completed comparator,

$$e^{i\Theta_{\text{total}}} = 1. \tag{37}$$

Equation (37) does not imply $|Z^{(1)}| = 1$. A fuzzy, positive support modulus may remain after phase closure.

6.5 No new poles

If J is restricted to $\operatorname{im} D$ or appears only as a constrained auxiliary variable, its correlators inherit their analytic structure from the phase field. No second LSZ sector is introduced. A later continuum derivation must verify this by inverting the full quadratic form, but the finite complex does not by itself postulate a new particle.

7 Auditable finite models

7.1 The unweighted four-cycle

For the four-cycle with unit stiffness, choose

$$B = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix}. \tag{38}$$

Then $H_0 = B^T B$ and $H_1 = BB^T$ both have spectrum

$$\{0, 2, 2, 4\}, \qquad \det{}' H_0 = \det{}' H_1 = 16. \tag{39}$$

The phase kernel is the constant mode; the current kernel is uniform circulation around the loop.

7.2 A weighted graph with two independent cycles

Take four vertices, the square boundary, one diagonal, and

$$K = \mathrm{diag}(1.0, 1.7, 0.6, 2.2, 1.3). \tag{40}$$

For the incidence matrix listed in Appendix B, direct diagonalization gives

$$\mathrm{spec}(H_0) = \{0, 2.3061116264, 5.0121639615, 6.2817244121\}, \tag{41}$$
$$\mathrm{spec}(H_1) = \{0, 0, 2.3061116264, 5.0121639615, 6.2817244121\}. \tag{42}$$

The common primed determinant is 72.608. The graph is connected and has cycle rank $E - V + 1 = 2$, hence $\dim\ker H_0 = 1$, $\dim\ker H_1 = 2$, and $\mathrm{ind}\, D = -1$.

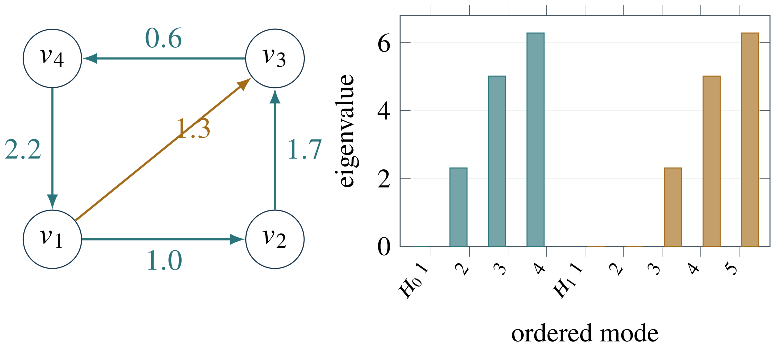

Figure 1: Weighted phase-current complex. The three positive eigenvalues agree exactly to numerical precision; the current sector retains the two independent cycle zero modes. Arrow reversal changes only the auxiliary representation.

7.3 Twisted Klein-bottle return topology

Use the one-vertex cellular model

$$\pi_1(K_2) = \langle a, b \mid aba^{-1} = b^{-1} \rangle \tag{43}$$

and the one-dimensional local system

$$\rho(a) = 1, \qquad \rho(b) = -1. \tag{44}$$

Fox differentiation of $r = aba^{-1}b$ gives, after evaluation,

$$\partial_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \qquad \partial_1 = \begin{pmatrix} 0 & -2 \end{pmatrix}, \qquad \partial_1\partial_2 = 0. \tag{45}$$

Both maps have rank one, so

$$H_0(K_2; L_\rho) = H_1(K_2; L_\rho) = H_2(K_2; L_\rho) = 0. \tag{46}$$

The local system is acyclic.

7.4 Unit torsion as a bookkeeping baseline

With standard Hermitian metrics,

$$\Delta_0 = 4, \qquad \Delta_1 = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, \qquad \Delta_2 = 4. \tag{47}$$

The Ray–Singer-type torsion is

$$\log T = \frac{1}{2}\sum_{q=0}^{2}(-1)^q q \log \det{}' \Delta_q = 0, \qquad T = 1. \tag{48}$$

Established / auditable result

The chosen twisted K_2 example lifts all cohomological return zero modes and leaves unit torsion. It therefore supports the revised interpretation particularly well: the seam is a consistent non-physical return bookkeeping device, not an automatically observable topological energy sector. Other local systems may leave non-trivial residues, but such residues must be calculated rather than presumed.

8 Uncertainty-local Euclideanization and Woit's reconstruction problem

8.1 A scale-dependent local definition

Let $U \subset M_4 \setminus \Sigma$ be a patch away from physical defects or source seams, and let k be the characteristic fluctuation wavenumber. Define the local regulator loops

$$\mathcal{L}_k(U) = \left\{ \gamma \subset U \setminus \Sigma \,\middle|\, \mathrm{diam}_g(\gamma) \le ck^{-1} \right\}. \tag{49}$$

A sufficient working condition for an uncertainty-local Euclidean regulator patch is

$$\epsilon_U(k) = \max\left\{ k^{-1}\|\nabla \ln K\|_{L^\infty(U)}, k^{-2}\|\,\mathrm{Riem}(g)\|_{L^\infty(U)}, [k\,\mathrm{inj}_g(U)]^{-1} \right\} \ll 1, \tag{50}$$

combined with

$$\mathrm{Hol}_{L_\rho}(\gamma) = \mathbf{1} \quad \forall \gamma \in \mathcal{L}_k(U), \qquad \inf_{s\in[0,1]} \lambda'_{\min}(H_s|_U) > 0. \tag{51}$$

The last condition prevents the Lorentzian-to-elliptic contour deformation from crossing a zero mode. These equations sharpen the earlier schematic phrase "uncertainty-local."

8.2 Euclideanity is a regulator chart, not the primary ontology

The order of construction is

$$\begin{aligned}&\text{premetric neighbour phase flow} \longrightarrow \text{coherent reciprocal comparator family} \\ &\longrightarrow \text{operational Lorentzian } M_4 \longrightarrow \text{local Euclidean one-loop chart.}\end{aligned} \tag{52}$$

A global Euclidean Separverse is neither assumed nor required. Caustics, defects, long holonomy loops, and support-history changes may obstruct a single global continuation.

8.3 Woit and Wistor

Woit's analysis begins from a Euclidean holomorphic chiral theory and asks how a conjugation can reconstruct a Minkowski state space [7]. The present construction uses this as a later consistency question: after M_4 has emerged, one still has to understand positivity and the relation between Euclidean regulators and Lorentzian observables. But the Wistor engine does not infer M_4 from a pre-existing $\mathbb{CP}^3$ ambient or from fundamental handedness.

> Woit studies how a chosen Euclidean-holomorphic theory may acquire a physical Lorentzian interpretation. Wistor asks what neighbour-mediated phase-current structure can generate the need for both Lorentzian measurement and local Euclidean regulation in the first place.

9 Anomalies, chirality, and CPT after the revision

9.1 What ordinary supersymmetry does and does not cancel

In conventional supersymmetric field theories, boson–fermion loop signs can soften ultraviolet terms. Chiral gauge anomalies are different: they are determined by the chiral fermion representation content. Scalar superpartners do not cancel a Weyl-fermion triangle anomaly. A partnerless ΦBSU model can therefore retain ordinary Standard-Model anomaly cancellation if its emergent fermion spectrum has the same anomaly-free representation structure.

9.2 The emergent M_4 anomaly audit

For a four-dimensional chiral gauge theory, the local anomaly data are encoded in the six-form

$$I_6 = \left[\widehat{A}(TM_4)\ \mathrm{ch}_R(F)\right]_6 . \tag{53}$$

A consistent emergent gauge theory must satisfy $I_6 = 0$ for the gauged redundancies, or provide a separately derived compensating mechanism. Global anomalies require the corresponding mod-two index, eta-invariant, or determinant-line holonomy calculation.

Conditional statement

Reciprocal phase closure is compatible with anomaly freedom because an anomalous gauge loop would obstruct a globally consistent completed comparator. But equation (12) is not itself a computation of I_6 or of a global determinant-line phase. The anomaly audit remains an ordinary M_4 consistency gate.

9.3 Why the seam is not an automatic anomaly reservoir

A bookkeeping seam may change local phase representatives. It does not automatically carry the opposite chiral determinant or an anomaly inflow action. Such a role would require an explicit coupling and a quantized variation. The revision therefore removes the earlier idea that orientation reversal or paired Klein seams automatically cancel local and global anomalies.

9.4 CPT as an emergent statement

Before the Lorentzian chart there is no spatial parity operator, no selected time orientation, no Weyl handedness, and no physical charge-conjugation convention. Once a local Lorentzian field algebra satisfies the usual locality, unitarity/positivity, spectrum, and spin-statistics assumptions, CPT is expected as a consistency property of that emergent sector. It is not imposed on the premetric cochain complex.

10 The $\sqrt{2}$-separated fuzzy support ambient

10.1 Scale support without a particle tower

The wider programme uses a frozen support hierarchy

$$L_n = L_0\Lambda^n, \qquad \Lambda = \sqrt{2}. \tag{54}$$

In the present essay this is an explicit model construction, not a consequence of the phase-current theorem. The key revision is how the hierarchy is interpreted. The label n is a support or memory scale, not a new particle species. A fuzzy spectral density can be written schematically as

$$\rho_{\mathrm{amb}}(\mu^2) = \sum_n w_n\, K_\sigma\!\left(\ln\frac{\mu}{\mu_0} - n\ln\sqrt{2}\right)\rho_n(\mu^2), \tag{55}$$

where the kernels overlap. The no-particle-tower condition is the absence of isolated positive-residue terms $Z_n\delta(\mu^2 - m_n^2)$ attributable to independent ambient fields.

10.2 Exact phase closure with fuzzy compensation partition

For every support history λ drawn from equation (55), equation (17) holds. What remains fuzzy is the decomposition

$$\Delta\theta_{\mathrm{comp}}(\lambda) = \Delta\theta_{\mathrm{state}}(\lambda) + \Delta\theta_{\mathrm{mom}}(\lambda) + \Delta\theta_{0\mathrm{f}}(\lambda), \tag{56}$$

not the total completed phase. This is the precise form of “cancellation without erasure”: the phase defect disappears from the completed comparator, while the compensating state and motion remain.

10.3 Holonomy resonance without a mass resonance

Because $L_{n+2} = 2L_n$, a useful return map is

$$\mathscr{R}_2 : U_n(\gamma) \longmapsto U_{n+2}(\gamma). \tag{57}$$

A holonomy resonance is a periodic orbit

$$\mathscr{R}_2^q[U_n(\gamma)] = U_n(\gamma), \tag{58}$$

not a pole at a new mass. It may appear as enhanced coherence in a Wilson-loop-like comparator, a finite form factor, a support-memory response, or a zero-fibre energy redistribution.

10.4 Bosonic zero-fibre energy

The factor U_{0f} in equation (12) denotes the proposed bosonic support response of the non-asymptotic null or zero-fibre sector. It is not assumed to be a free bosonic particle. To become a referee-level construct, the programme must supply an action, a stress tensor or energy functional, a coupling to the comparator, and a demonstration that integrating the sector does not introduce an unwanted LSZ pole.

Open derivation target

Derive a local or quasi-local functional $E_{0f}[\alpha, J, \text{support}]$ such that its variation supplies the required compensation in equations (15) and (16), while its spectral density remains broad and pole-free.

11 Relation to supersymmetry and cohomological field theory

The revised use of the word *hypersymmetry* should be compared carefully with supersymmetry.

Feature	Conventional supersymmetry	Revised ΦBSU hypersymmetry
Primitive pairing	Bosonic and fermionic fields in a supermultiplet	Phase fluctuation and its exact neighbour-current response
Algebra	Super-Poincare or related superalgebra	Cochain map D and linear degree grading
Loop sign	Fixed by statistics and supersymmetry	Not fixed by isospectrality; must come from the derived measure
Physical partners	Usually additional on-shell states	None required; current sector may be auxiliary/exact
Unpaired sector	Supersymmetric ground states / index	Constants, cycles, seams, twisted cohomology, support residue
Anomalies	Checked from chiral fermion content; SUSY organizes multiplets	Checked in emergent M_4 from actual chiral content
All-loop protection	Possible through supercharges and holomorphy	Not established; present claim is one-loop quadratic pairing

The nearest mathematical relatives are Hodge complexes, BRST resolutions, topological field theory, and analytic torsion. The nearest physical aspiration is not a hidden low-energy SUSY spectrum but a constrained one-loop reduction in which exact bulk fluctuations are redundant while topological or support information remains.

12 The unification horizon, now with a cleaner hierarchy of roles

12.1 Identity and curvature

The identity/curvature split in equation (2) remains the most economical bridge to gauge theory. The revision adds that A_{id} should not be asked to carry radiative energy. Relative phase bookkeeping closes only together with local curvature, matter, momentum, and support responses.

12.2 The $3 \otimes \bar{3} = 1 \oplus 8$ colour node

The quark–gluon companion note uses

$$\mathrm{End}(\mathbb{C}^3) = \mathbb{C}I \oplus \mathfrak{sl}(3, \mathbb{C}) \tag{59}$$

as a compact operational chart of a 3^2 holonomy node [6]. This representation identity is established mathematics; its identification with a background-independent interior node is programme-specific. It is logically independent of the phase-current theorem, but both may eventually descend from a common neighbour-complex construction.

12.3 Vacuum-structure numbers and matter amplitudes

The fine-structure and charged-lepton companion note proposes a 2–3–5 information hierarchy and a quadratic measurement threshold [5]. These numerical constructions should remain in the unification horizon unless a variational or representation-theoretic derivation connects them to κ_Φ and the reciprocal comparator. Their value in the present essay is as independent falsification modules, not as proof of hypersymmetry.

12.4 Part II support and the ambient

Part II treats the $\sqrt{2}$ ladder as a frozen sourcewise support hierarchy and reports a range of working closures and benchmark tests [3, 4]. The present revision supplies a conceptual constraint on any such support model: the support may redistribute compensation and remain fuzzy, but completed M_4 standards must close exactly and the ambient must not be reinterpreted as an arbitrary hidden particle halo.

12.5 What a genuine unified derivation would accomplish

A successful κ_Φ derivation would need to produce, from one action or measure:

1. the premetric phase-current complex and its positive local Hessian;
2. a stable rank-four hyperbolic coarse principal symbol;
3. reciprocal comparator closure and the local conservation law;
4. the gauge identity/curvature split and matter representations;
5. a canonical one-loop measure with no unwanted poles;
6. ordinary M_4 anomaly consistency and a viable low-energy spectrum;
7. the $\sqrt{2}$ support hierarchy or a reason it is only an effective approximation.

The revision deliberately does not pretend that one of these items proves the others.

13 Claim ledger

Table 1: Status of the main statements after the reciprocal-closure revision.

Claim	Status	What is presently established	Next decisive test
$D^\dagger D$–$DD^\dagger$ non-zero pairing	Established mathematics	Singular-value theorem; finite models reproduce it	Continuum domain and convergence analysis
Orientation independence	Established mathematics	$D \mapsto SD$ leaves H_0 fixed and conjugates H_1	Randomized finite-complex tests
Reciprocal total phase 1	Explicit model postulate	Written as equation (12); clarifies physical interpretation	Derive from a local action and conservation law
Phase shift implies compensating movement	Explicit model postulate	Channel decomposition is clear	Construct $T^{\mu\nu}_{0f}$ and verify equation (16)
Holonomy seams are non-physical transition data	Explicit construction	Cech/local-system representation and seam-move invariance	Show equivalence under changes of cover in a continuum model
Twisted K_2 baseline	Established finite result	Acyclic complex, $T = 1$ for $\rho(a) = 1, \rho(b) = -1$	Explore physically motivated local systems without assuming residue
Naive first-order measure	Established finite/formal calculation	Gives only $[\det{}' H_0]^{-1/2}$	None; this route is ruled out as a cancellation by itself
Physical determinant cancellation	Conditional / open	Requires canonical Jacobian or BRST/BV measure	Derive all fields, statistics, zero-mode volumes, and regulator
No new poles	Conditional	Natural if current is exact/auxiliary	Invert full continuum quadratic form and inspect LSZ poles
Uncertainty-local Euclideanization	Explicit criterion	Equations (50)–(51) sharpen the patch definition	Prove regulator equivalence and spectral-gap stability
Gauge anomaly freedom	Open ordinary M_4 gate	No longer attributed automatically to seams	Compute actual emergent chiral representation anomalies
CPT emergence	Conditional	Expected only after a local Lorentzian QFT algebra exists	Verify locality, positivity, spectrum, spin-statistics
$\sqrt{2}$ fuzzy ambient	Explicit programme construction	Scale ladder and pole-free interpretation are stated	Derive support spectral density and observational response
Bosonic zero-fibre compensation	Open	Role in closure is specified	Derive energy functional, coupling, and pole-free stress response
Emergent M_4	Open programme target	Operational reciprocal invariant stated	Derive four-dimensional hyperbolic coarse geometry from κ_Φ
Gauge / matter / unification horizon	Open	Representation compatibilities and numerical modules exist	Common action, couplings, RG flow, and blind tests

14 Falsification and the next research cycle

14.1 Gate 1: finite-complex reproducibility

Release the incidence matrices, stiffness matrices, spectra, heat traces, twisted boundary maps, and torsion calculation. Randomize all edge orientations. Any failure of the stated spectral theorem under its hypotheses is a mathematical stop condition.

14.2 Gate 2: reciprocal closure in a dynamical model

Construct the smallest local toy action containing a signal field, a material degree of freedom, momentum exchange, and a zero-fibre support variable. Demonstrate equation (12) on solutions and equation (16) at the Noether level. If the total phase closes only after ensemble averaging, the strong version of the model is false.

14.3 Gate 3: the physical one-loop measure

Specify all fields and statistics. Derive the measure rather than choosing a determinant quotient. The answer may be no cancellation, a trivial exact-sector resolution, or a non-trivial residue. Each is informative. The mechanism fails as a stabilization claim if no canonical opposite determinant weight exists.

14.4 Gate 4: pole audit

Compute the full propagator matrix of the continuum quadratic model. The ambient, current, and zero-fibre channels must not produce unobserved isolated positive-residue poles unless the theory intentionally predicts new states.

14.5 Gate 5: emergent geometry and local Euclidean regulator

Show that a refining family of complexes converges to a rank-four hyperbolic operator. Then verify the uncertainty-local conditions and compare heat-kernel, zeta, and contour regulators. A global Euclidean continuation is not required.

14.6 Gate 6: ordinary gauge consistency

List the emergent Weyl fermions and compute all local and global gauge anomalies. The seam cannot be invoked as a cancellation unless a quantized compensating action is explicitly derived.

14.7 Gate 7: fuzzy ambient response

Construct a broad spectral density of the form (55), demonstrate exact reciprocal closure for sampled support histories, and search for holonomy-coherence resonances without mass poles. Vary the packet width and regulator; a topological or structural prediction must survive.

14.8 Gate 8: independent phenomenology

Treat the fine-structure, lepton, quark, galaxy-support, and early-seeding modules as separate tests with machine-readable inputs and predeclared predictions. Agreement in one module must not compensate statistically or rhetorically for failure in another.

15 Conclusion

The reciprocal phase-closure statement removes a large amount of unnecessary conceptual machinery. A measurable phase shift is real, but it is not a free defect floating on a second sheet. It is one entry in a completed event whose other entries are material-state change, momentum transfer, local curvature energy, and bosonic zero-fibre support. The total event closes to phase one.

This makes the status of the holonomy seam much clearer. The seam is a transition function, return rule, or branch cut in phase bookkeeping. It may encode winding and twisted boundary conditions, but it does not need to be a physical mirror surface. The explicit twisted K_2 example is acyclic and has unit torsion, showing that a non-orientable return rule can remove bookkeeping zero modes without leaving an additional physical sector.

The one-loop core remains unchanged and becomes easier to state. The phase Hessian $D^\dagger D$ and exact-current Hessian $DD^\dagger$ share their positive spectrum. This is the precise content of hypersymmetry in the revision. It offers a route to partnerless one-loop organization, but it does not by itself supply a determinant minus sign. The naive commuting first-order measure has now been calculated explicitly and gives no cancellation.

Three tasks therefore remain distinct. First, derive the functional measure governing the determinant modulus. Second, derive reciprocal phase closure and the compensating stress-energy law. Third, construct an emergent M_4 field theory that passes ordinary anomaly, positivity, and CPT consistency tests. The $\sqrt{2}$ ambient may remain fuzzy because exact closure constrains the completed product, not the microscopic partition of support.

Revision principle

proved phase-current pairing $\longrightarrow$ derived measure and reciprocal conservation $\longrightarrow$ emergent M_4 gauge theory $\longrightarrow$ independently testable unification

The result is a simpler and more falsifiable programme. Phase error cancels; compensating movement remains. The seam keeps the books; the local channels carry the physics.

A Notation and status vocabulary

Symbol / term	Meaning
C	Premetric neighbourhood or cell complex
$C^p(C)$	p-cochains on the complex
d_0	Neighbourhood coboundary from vertex phases to edge differences
K	Positive linearized stiffness operator on one-cochains
$D = K^{1/2} d_0$	Phase-to-current amplitude map
$H_0 = D^\dagger D$	Phase-fluctuation Hessian
$H_1 = DD^\dagger$	Exact-current Hessian
$\mathscr{H}_{M_4}$	Completed reciprocal comparator holonomy including response channels
$U_{0\mathrm{f}}$	Proposed bosonic zero-fibre support response, not assumed to be a free particle
K_2	Non-orientable return topology / twisted local-system bookkeeping
Wistor	Premetric phase-current engine; not Penrose twistor space
M_4	Emergent operational Lorentzian meter chart
uncertainty-local	Scale-dependent patch satisfying equations (50)–(51)
Established result	Follows from stated mathematics or an explicit finite calculation
Explicit construction	Fully specified programme element not forced by the theorem

Conditional statement	Follows only if a stated measure, matching, or emergence condition holds
Open target	Missing derivation or empirical test

B Matrix data for the weighted example

The incidence and stiffness matrices used in Figure 1 are

$$B = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{pmatrix}, \qquad K = \mathrm{diag}(1, 1.7, 0.6, 2.2, 1.3). \tag{60}$$

The positive eigenvalues are

$$2.3061116263805, \qquad 5.0121639615412, \qquad 6.2817244120783, \tag{61}$$

with product 72.608. The cycle rank is $5 - 4 + 1 = 2$.

C Twisted K_2 calculation and torsion

For $r = aba^{-1}b$, the Fox derivatives may be written

$$\frac{\partial r}{\partial a} = 1 - aba^{-1}, \qquad \frac{\partial r}{\partial b} = a + aba^{-1}. \tag{62}$$

Using the relation $aba^{-1} = b^{-1}$ and the representation $\rho(a) = 1$, $\rho(b) = -1$ gives equation (45). The chain complex

$$0 \longrightarrow \mathbb{C} \xrightarrow{\partial_2} \mathbb{C}^2 \xrightarrow{\partial_1} \mathbb{C} \longrightarrow 0 \tag{63}$$

is exact. The Laplacian determinants are

$$\det \Delta_0 = 4, \qquad \det \Delta_1 = 16, \qquad \det \Delta_2 = 4. \tag{64}$$

Thus

$$\log T = -\frac{1}{2} \log 16 + \log 4 = 0. \tag{65}$$

Because the result is unity, it is independent of whether a convention reports T or T^{-1}.

D The first-order measure calculation in finite dimensions

Let $\eta \in \mathbb{R}^V$, $J, \lambda \in \mathbb{R}^E$, with $K > 0$. Regularize zero modes by restricting η to $(\ker D)^\perp$. Then

$$Z = \int \mathrm{d}\eta\, \mathrm{d}J\, \mathrm{d}\lambda\, \exp\left[-\frac{1}{2} J^T K^{-1} J - \mathrm{i}\lambda^T (J - KB\eta)\right] \tag{66}$$

$$\propto \int \mathrm{d}\eta\, \mathrm{d}J\, \delta(J - KB\eta) \exp\left[-\frac{1}{2} J^T K^{-1} J\right] \tag{67}$$

$$= \int \mathrm{d}\eta\, \exp\left[-\frac{1}{2} \eta^T B^T K B \eta\right] \tag{68}$$

$$\propto [\det{}'(B^T K B)]^{-1/2}. \tag{69}$$

No $\det H_1$ factor is generated. If one instead declares J to be the fundamental flat coordinate on $\operatorname{im} D$, the Jacobian in equation (34) changes the normalization; that is a different measure choice and must be justified by the underlying theory.

E Minimal reproducibility pseudocode

```
input: oriented incidence matrix B (E x V)
      positive stiffness matrix K (E x E)

D = sqrtm(K) @ B
H0 = transpose(D) @ D
H1 = D @ transpose(D)

lambda0 = eigvalsh(H0)
lambda1 = eigvalsh(H1)
assert positive_spectrum(lambda0) == positive_spectrum(lambda1)

for each diagonal edge-sign matrix S:
   Dp = S @ D
   H0p = transpose(Dp) @ Dp
   H1p = Dp @ transpose(Dp)
   assert H0p == H0
   assert spectrum(H1p) == spectrum(H1)

# twisted Klein baseline
partial2 = [[2], [0]]
partial1 = [[0, -2]]
assert partial1 @ partial2 == 0
assert homology_dimensions(partial2, partial1) == [0, 0, 0]
assert ray_singer_torsion(partial2, partial1) == 1
```

F Revision map from the exploratory draft

Exploratory language	Revised meaning	Status
Pseudo-mirror sheet	Auxiliary cover or local return representation	Removed from physical ontology
Antilinear reflection R	Not the hypersymmetry generator	Reserved, if needed, for later reconstruction conventions
$P_+ = (1+R)/2$ **physical projection**	No longer used as the formal one-loop mechanism	Replaced by exact-current complex and operational readout
Mirror determinant cancellation	Positive spectral pairing only	Minus sign remains a measure problem
Klein seam cancels anomalies	Seam is transition/local-system bookkeeping	Anomalies audited in emergent M_4
Non-physical mirror poles	No independent mirror propagator is introduced	No-pole claim tied to auxiliary/exact status
Phase mismatch across sheets	Completed reciprocal phase closes with material and support response	Central revision postulate

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