IDENTITY BEFORE ENTROPY

A history-refinement framework, continuation multiplicity, and the ΦBSU holonomy bridge

ORDER-THEORETIC CORE: PULLED-BACK PARTITION REFINEMENT
CONTINUATION MULTIPLICITY: $m_{21} \geq 2$ UNDER STRICT REFINEMENT
0D BOUNDARY: $\mathbb{C}1$ IF NO ZERO-SUPPORT SECTOR SURVIVES

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Scope: the quantitative fate of physical information - partitions, continuation multiplicities, measures, and capacities.

Contents

Main result

The memorandum separates three carriers of physical information: distinctions in a complete causal-history domain, records readable at a later instant, and the complete algebra of a Cauchy slice. They do not obey one growth law. History refinement is order-logical, persistence of readable records is a dynamical arrow condition, and invariance of the complete algebra is a conditional no-go in the regular QFT regime.

Let $X(D)$ be the set of raw descriptions on a domain, $H(D)$ its gauge quotient, and $C(D)$ the family of admissible physical comparators. A raw prefix restriction is defined first on $X(D_2)\to X(D_1)$. Comparator support fidelity must show that it descends to a well-defined physical map

$$r_{21} : H(D_2) \to H(D_1).$$

If every admissible earlier prefix has at least one continuation, r_{21} is surjective and the comparator pullback

$$r_{21}^{*} : C(D_1) \hookrightarrow C(D_2),\ r_{21}^{*}(f)=f\circ r_{21}$$

is injective. Partitions on different carriers are not compared literally. The earlier equivalence relation is first pulled back to $H(D_2)$:

$$h \equiv_{21} h' \Leftrightarrow r_{21}(h) \equiv_1 r_{21}(h').$$

If the later instrument family contains the pullbacks of the earlier instruments, its relation satisfies $\equiv_2\subseteq\equiv_{21}$. Theorem A is therefore an analytic consistency framework. Its physics lies in whether r_{21}, surjectivity, quotient descent, comparator fidelity, and genuine novelty can actually be derived from ΦBSU dynamics.

Identity before entropy follows from a second order step. At an absolute comparator-free boundary, every unsupported raw label is gauge if no nontrivial closed comparator exists. If the gauge action is transitive there and no topological, edge, or superselection center survives, the invariant functions are constant:

$$C_0^+ = \mathbb{C}1,\ |\mathrm{Spec}(C_0^+)|=1,\ \mathrm{Cap}(C_0^+)=0 \text{ bits}.$$

Only the first nonconstant comparator creates a binary or richer distinction. A probability distribution and an entropy can be defined only after an alternative algebra exists. The logical type order is therefore

$$\text{identity / sameness quotient} \to \text{distinction} \to \text{state or probability} \to \text{entropy}.$$

Low entropy does not reverse this implication. A pure zero-entropy state may live on a large alternative space.

The framework yields four independent statements:

1. The **order of history distinctions** does not coarsen in a prefix-consistent, comparator-faithful model. No record arrow is required.
2. The **reservoir of currently accessible records** is non-decreasing only if earlier readouts persist or are recoverable. That is the open physical content of the record-arrow condition.
3. The **capacity of a complete Cauchy algebra** is invariant if the regular-regime theory implements a unitary time-slice isomorphism. This is a conditional low-energy no-go, not an established law at a Planck boundary.
4. **Continuation multiplicity** is at least two in some earlier prefix class if a one-class identity boundary develops a strict distinction refinement. This quantitative statement decides neither selection, probability, nor actualization.

ΦBSU adds a specific admission rule. The project sources admit a new information domain only when it carries an irreducible separation relation not represented in an earlier domain. Every admitted ΦBSU event must therefore make the prefix refinement strict. The proof burden is no longer the abstract question whether information grows. It is concrete: for every proposed update, exhibit a new gauge-invariant comparator, prove that it descends to the physical quotient, and pass the no-alias test.

1. Three information reservoirs

The term information reservoir must not denote three different objects.

Symbol	Object	What it measures	Growth status
$C^H(D,F)$	Comparator algebra or finite-instrument distinction partition of history prefixes	Which physically admissible histories can be distinguished from the content of the whole domain	The pulled-back relation does not coarsen; strict at a non-aliasing event
$R^{acc}(\tau)$	Algebra of records physically readable at τ	Which earlier outcomes can be read now	Monotone only under record persistence, recoverability, or an arrow postulate
$A(\Sigma)$	Complete physical algebra of a Cauchy surface	All distinctions of the theory's slice state	Isomorphism-invariant in the time-slice/unitary regime

Entropy $S(\rho)$, relative entropy $D(\omega\|\varphi)$, algorithmic complexity $K^U(x)$, and capacity do not form a fourth common reservoir. They are different functions on different objects:

- $S(\rho)$ is a property of a state on an already selected algebra;
- $D(\omega\|\varphi)$ measures the asymmetric distinguishability of two hypotheses;
- $K^U(x)$ depends on a coding and a universal machine up to an $O(1)$ term;
- partition refinement is an order relation prior to these numerical measures.

The primary object of this memorandum is the distinction structure of history prefixes. The accessible-record reservoir is treated by a separate conditional theorem, so later readability is not conflated with the content of the complete domain.

2. Paradigm-neutral hypotheses

The following conditions are not justified by importing fixed-background QFT. They form the conditional minimal framework of Theorem A; the gates decide whether physics realizes them. H0-H3 yield non-coarsening. H4 yields the identity boundary. H5 yields strict ΦBSU refinement.

H0. Separative extensionality

Let $X(D)$ be the admissible raw-description set and $\sim_{g,D}$ the model's gauge equivalence. Physical histories are $H(D)=X(D)/\sim_{g,D}$. A name, coordinate label, or second inscription of an identical structure does not constitute a second event. The complete admissible comparator family is also assumed to separate the points of $H(D)$: if two physical histories agree under every gauge-invariant comparator, they belong to the same operational history.

For a finite instrument family $F\subset C(D)$, define the weaker distinction relation

$$h \equiv_{F,D} h' \Leftrightarrow f(h)=f(h') \text{ for every } f\in F.$$

The operational classes and their partition are

$$Q_F(D):=H(D)/\equiv_{F,D},\ \Pi_F(D):=\{\text{the } Q_F(D) \text{ blocks in } H(D)\}.$$

This is the finite-resolution distinction object used in the main theorem. It separates gauge sameness, complete operational extensionality, and the limited resolution of a selected instrument.

H1. Coherent raw-prefix structure

Causal domains form a partially ordered system. When $D_1 \subseteq D_2$, a later raw history has an earlier raw restriction

$$\dot{r}_{21} : X(D_2) \to X(D_1),$$
$$\dot{r}_{31} = \dot{r}_{21} \circ \dot{r}_{32}.$$

The restriction is defined on raw descriptions first. It descends to a physical map $r_{21}:H(D_2)\to H(D_1)$ only after the quotient-compatibility condition in H3 is verified.

This structure requires neither Lorentz symmetry, a unique microscopic front speed, an ADM foliation, nor even a background manifold. A shared acyclic causal order or event poset is sufficient.

H2. Extendibility of physical prefixes

Every physically admissible earlier prefix has at least one admissible continuation. Once r_{21} has descended to the quotient, it is surjective. If a history can terminate, termination is represented by an absorbing terminal symbol, so that the terminated history remains a valid later prefix rather than disappearing retroactively.

H2 does not say that an earlier measurement result remains readable. It prevents a model from retroactively declaring an already admissible past impossible unless an explicit global boundary condition or another non-cylindrical rule is introduced. A terminal symbol preserves a genuinely terminated history only if the terminal continuation is an admissible physical object of the theory; it cannot rescue a prefix that a global condition makes retroactively inconsistent. H2 is projective model consistency, not a thermodynamic arrow.

H3. Comparator support fidelity and quotient descent

Every comparator of the earlier domain remains a valid function of the later raw history by pullback:

$$\dot{r}_{21}^{*}(f) = f \circ \dot{r}_{21}.$$

The pullback must be gauge-invariant on the later domain. In particular,

$$x \sim_{g,D_2} y \Rightarrow \dot{r}_{21}(x) \sim_{g,D_1} \dot{r}_{21}(y),$$

so $r_{21}([x])=[\dot{r}_{21}(x)]$ is well defined on the gauge quotients. For selected instrument families F_1 and F_2, the pullback condition $r_{21}^{*}F_1 \subset F_2$ also gives comparator descent:

$$h \equiv_{F_2,D_2} h' \Rightarrow r_{21}(h) \equiv_{F_1,D_1} r_{21}(h').$$

Hence there is a well-defined induced map on operational classes,

$$rQ_{21}:QF_2(D_2)\to QF_1(D_1),\ rQ_{21}([h]_2)=[r_{21}(h)]_1.$$

The representation must also be faithful: distinct physical comparators must not be identified merely by the encoding. Gauge descent and comparator descent are logically prior to testing surjectivity.

H3 distinguishes a full-history model from a coarse-graining. A representation that deliberately forgets old components may display a decreasing reservoir, but it is not a faithful representation of the same C^H object.

H4. Apical gauge irreducibility and closure locality

The initial boundary is not assumed to be a regular point of a manifold. It is defined by a shrinking filter $\{D_\lambda\}_{\lambda\to 0}$ of causal domains or by an initial object of the history category. Every nonconstant physical comparator requires a nontrivial closed history or at least positive separative support. The boundary has no such support.

All raw labels unsupported by a closed comparator are gauge. If the gauge action is transitive at the boundary and no topological, edge, or superselection center survives, the invariant algebra is $\mathbb{C}1$.

This is stronger and more accurate than extrapolating Verch's local definiteness theorem. Verch's theorem concerns Hadamard states at interior points of a regular manifold. H4 is a constitutive condition on the ΦBSU ultraviolet boundary and must be derived, or refuted, from the theory's own closure structure.

H5. Non-aliasing event novelty

A new physical event is admitted only if the later domain contains a comparator g and two continuation histories h,h' such that

$$r_{21}(h)=r_{21}(h'), \text{ while } g(h)\neq g(h').$$

The new distinction is then not a repeated name, divisibility label, holonomy label, or projection class already present. H5 does not claim that every interval of continuous coordinate time contains an event. It makes event-indexed refinement strict.

Status of the hypotheses

Condition	Character	Meaning of failure
H0	Operational definition of physical identity	Primitive haecceity: physically indistinguishable labels are nevertheless counted as distinct existences
H1	Coherence of causal histories	No common prefix order, as in a strong indefinite-order branch
H2	Projective consistency and extendibility	A future boundary condition invalidates an earlier admissible prefix, or a history genuinely vanishes from the model
H3	Quotient descent and representational fidelity	Restriction is ill-defined on physical classes, or the model coarse-grains or forgets part of the history
H4	Identity condition of the ΦBSU ultraviolet boundary	A primitive topological, edge, or superselection distinction remains at the boundary
H5	ΦBSU event-admission rule	Every fiber has $m_{21}=1$, or the update is an operational alias

3. Theorem A: pulled-back refinement of history distinctions

Prefix-extension lemma

Let $r_{21}:H(D_2)\to H(D_1)$ be a surjective prefix restriction already shown to be well defined on the physical quotients. Let $C(D)$ be the algebra of finite-outcome physical comparators or the corresponding distinction lattice. Then

$$r_{21}^{*}(f)=f\circ r_{21}$$

defines an injective pullback.

Proof. If $f\neq g$ in $C(D_1)$, there is $h_1\in H(D_1)$ with $f(h_1)\neq g(h_1)$. By surjectivity, some $h_2\in H(D_2)$ satisfies $r_{21}(h_2)=h_1$. Hence $r_{21}^{*}(f)(h_2)\neq r_{21}^{*}(g)(h_2)$, and therefore $r_{21}^{*}(f)\neq r_{21}^{*}(g)$. ■

Theorem A. Foliation-covariant pulled-back partition refinement

Assume H0-H3. Choose instrument families $F_1 \subset C(D_1)$ and $F_2 \subset C(D_2)$ such that $r_{21}^* F_1 \subset F_2$. The relation $\equiv F_1, D_1$ lives on $H(D_1)$, so first pull it back to $H(D_2)$:

$$h \equiv_{21} F_1\ h' \Leftrightarrow r_{21}(h) \equiv F_1, D_1\ r_{21}(h').$$

The later distinction relation is then contained in the pulled-back earlier relation,

$$\equiv F_2, D_2 \subseteq \equiv_{21} F_1,$$

or, in partition notation,

$$\Pi F_2(D_2) \succeq r_{21}^{-1} \Pi F_1(D_1).$$

Proof. If $h \equiv F_2, D_2 h'$, every comparator in F_2 gives the same value on h and h'. Since $r_{21}^* F_1 \subset F_2$, in particular $f(r_{21}h) = f(r_{21}h')$ for every $f \in F_1$. Hence $h \equiv_{21} F_1 h'$. ■

H3 makes $rQ_{21}: QF_2(D_2) \to QF_1(D_1)$ well defined, and H2 makes it surjective. Hence, for finite instrument families,

$$|QF_1(D_1)| \leq |QF_2(D_2)|,\ CF(D) = \log_2 |QF(D)|.$$

Thus $CF_1(D_1) \leq CF_2(D_2)$. In an infinite setting, the primary claim is inclusion of relations: cardinalities can remain equal despite a proper refinement.

Modality and physical content

Theorem A is an order-logical consistency framework. Once H0-H3 and the inclusion of instrument families are granted, refinement is analytic. Its physical weight lies in the gates: the raw map $\dot{r}_{21}$, its descent to the gauge quotient, surjectivity of the quotient map, and comparator support fidelity must all be derived. ΦBSU-specific content begins only when H5 is supplied with a non-aliasing event operator.

The theorem does not claim that a trace created by an event remains readable in a later apparatus. It compares the complete history in D_1 with the complete history in D_2. The earlier prefix belongs to the later history; this is an order property, not decoherence dynamics.

A time-reversed history forms its own prefix chain relative to the chosen orientation. The theorem alone does not select which end of the causal orientation is called the origin. A cosmological arrow still requires a boundary condition.

Why type III does not empty the theorem

The primary content is the pulled-back order of finite operational partitions, not a finite dimension assigned to an unsmeared ultraviolet algebra. For von Neumann algebras, Araki relative entropy remains available in type-III cases. Type-I interpolating factors supplied by a justified split property may serve as finite-resolution approximations, but they are not premises of Theorem A. No universal finite CF value is assumed for a bare type-III_1 local net.

Strict refinement

If H5 holds between D_1 and D_2 and the novelty comparator belongs to F_2, the inclusion is proper:

$$\Pi F_2(D_2) \succ r_{21}^{-1} \Pi F_1(D_1).$$

A plateau occurs exactly when $\equiv F_2, D_2 = \equiv_{21} F_1$, equivalently when the induced class map rQ_{21} is bijective. Bijectivity of the raw prefix map alone is not sufficient for a limited instrument family. The exact content is:

- the order does not coarsen along any faithful causal foliation;
- refinement is strict at every demonstrated non-aliasing event;
- event-free intervals may remain plateaus.

Foliation independence

A foliation is an order-preserving chain $F:T\to\mathcal{D}$ in the poset of causal domains. Along every such chain, the pulled-back partitions form a monotone system. If the comparators and prefix maps form a genuinely functorial directed system, two cofinal chains produce isomorphic inductive comparator limits. Their instantaneous numerical capacities may differ because they sample different domains, but neither can reverse the refinement order.

This is the defensible content of the claim across all faithful modelings. A modeling map must preserve prefix and comparator morphisms. A non-faithful coarse-graining models a different object.

4. Theorem B: identity before entropy

The initial boundary without an assumed manifold point

The initial boundary is not assigned the status of an interior spacetime point. Let $\{D\lambda\}\lambda>0$ be a shrinking filter of causal domains approaching the theory's operational initial object. First form the inductive limit $C\infty$ of the comparator algebras under the prefix injections, with canonical embeddings $j\lambda$. Only inside this common ambient algebra may the boundary comparators be defined:

$$C_0^+ := \bigcap\lambda>0\ j\lambda(C(D\lambda)) \subseteq C\infty.$$

Equivalently, one may use the projective limit of history prefixes. A geometric GKP/TIP/TIF completion may be constructed later, but is not smuggled into the theorem.

Theorem B. Apical identity algebra

Assume H0-H4. Every gauge-invariant comparator surviving at the boundary is constant, and

$$C_0^+=\mathbb{C}1.$$

Proof sketch. By closure locality, a nonconstant comparator requires a nontrivial closed history or positive separative support. Neither exists at the initial boundary. The remaining raw labels belong to one gauge orbit by H0 and H4. Invariant functions under a transitive gauge action are constant, so $C_0^+=\mathbb{C}1$. ■

The algebra $\mathbb{C}1$ has one normalized state. It is the initial object in the category of unital complex $*$-algebras with unital complex-linear $*$-homomorphisms: for every algebra A there is exactly one such morphism $\lambda\mapsto\lambda 1\_A$. Its operational capacity is zero bits.

Logical type order of entropy

Entropy first requires alternatives and a state or probability measure over them. The boundary has one equivalence class and its only normalized distribution is $p(*)=1$. Its Shannon and von Neumann entropies therefore vanish trivially.

The converse fails. An N-qubit algebra $M_{2^N}(\mathbb{C})$ has N bits of orthogonal capacity, yet every pure state has $S=0$. A pure state may also require a long algorithmic description. Hence

$$\text{identity} \Rightarrow \text{zero capacity} \Rightarrow \text{zero entropy},$$

but zero entropy implies neither zero capacity nor identity.

The exact thesis is stronger than the Past Hypothesis but narrower than a claim about the universe's complete ontic initial state:

Identity is the type-theoretic beginning of physical distinction logic. If the same initial object is also the boundary of the cosmological history order, identity is additionally a temporal beginning. Low entropy is only the consequence evaluated on the unique state of that boundary algebra.

5. The ΦBSU holonomy bridge

The bridge is the critical path of the memorandum. Without an explicit morphism, ℂ1, holonomic structural identity, and the identity connection would remain three analogies sharing a name. The project sources support the following target construction.

5.1 Comparator path groupoid

Let $\mathrm{Path}_\Phi(D)$ be the graded groupoid of locally transported null-history paths in D. Its objects are comparison events and its morphisms are transported histories Γ. For every object x, the empty path ε_x is its unique identity morphism. Inversion is formal path inversion and does not by itself assert reversibility of physical time evolution. Part I states that global phase data become operational only when histories form a closed comparator. Closed morphisms $\Gamma:x\to x$ are therefore the relevant holonomy comparators, but a nonempty return with trivial holonomy is not the same morphism as ε_x.

Write the identity connection and its local phase lifts as

$$A_{id}=-i\Phi^{-1}d\Phi,\quad A_{id}|U_i=d\alpha_i.$$

If α is a globally single-valued real function, $\oint_\Gamma d\alpha=0$ on every closed path. Nonzero winding requires a local or multivalued lift or topology carried by transition data. Under these conditions the connection defines a lifted return number and, where applicable, a U(1) holonomy:

$$n_\Gamma=(1/2\pi)\oint_\Gamma A_{id},$$
$$\mathrm{Hol}_A(\Gamma)=\exp(i\oint_\Gamma A_{id}).$$

The return number is additive and the holonomy multiplicative:

$$n(\Gamma_2\circ\Gamma_1)=n\Gamma_2+n\Gamma_1,$$
$$\mathrm{Hol}_A(\Gamma_2\circ\Gamma_1)=\mathrm{Hol}_A(\Gamma_2)\mathrm{Hol}_A(\Gamma_1).$$

For the empty path, $n_\varepsilon=0$ and $\mathrm{Hol}_A(\varepsilon_x)=1$. If Φ is a globally circle-valued field and $\oint_\Gamma A_{id}=2\pi n$, the unit-charge Wilson number $\exp(i2\pi n)$ is always one and cannot distinguish returns. The physical comparator must then read a demonstrated gauge-invariant lifted winding, a seam-sensitive or P_+-projected character, Čech transition data, or another closure label.

The comparator algebra $W_\Phi(D)$ is generated only by quantities shown to be gauge-invariant and operationally readable, together with the $H^A(x,k,\Gamma)$ closure labels. A merely suggestive phase expression is not yet a physical comparator.

5.2 Canonical morphism

Every domain has a canonical unital morphism

$$\eta_D:\mathbb{C}\to W_\Phi(D),\quad \eta_D(\lambda)=\lambda 1_D.$$

This map does not identify the three meanings of identity. It states their exact relation:

- the algebraic unit 1_D is the constant character of the empty comparator path;
- A_{id} transports phase identity along paths;
- structural identity is a candidate winding, seam, or closure sector whose preservation under the selected history extension must be proved in CP4;
- new history information appears only when a winding, seam, projection, or closure comparator distinguishes history hypotheses.

If the absolute zero-dimensional seed has the trivial one-object path groupoid modulo gauge,

$$\mathrm{Path}_\Phi(D_0)/\mathrm{gauge}=\{\varepsilon\},\quad W_\Phi(D_0)=C^*(\{\varepsilon\})=\mathbb{C}1.$$

This is the target statement for the missing constitutive derivation. It does not reduce the number of assumptions by itself: $\mathrm{Path}_\Phi(D_0)/\mathrm{gauge}=\{\varepsilon\}$ is H4 in native groupoid language. CP3 must derive or refute it from ΦBSU closure dynamics.

5.3 Return history and persistent structural identity

In the same source chain, the centribuoyancy construction gives the local transport $q|U_i=d\alpha_i$, the holonomic return condition

$$\oint_\Gamma q_\mu \, dx^\mu=2\pi n,$$

and the return count $\tau_\Gamma=N_\Gamma T_\Gamma$. The phrase structural identity persists while its history grows return by return now has a testable target model. A history $h_N=(\Gamma_1,\dots,\Gamma_N)$ extends as $h_N\mapsto h_{N+1}$. Raw winding is additive under composition, so only a demonstrated sector projection, compensated return class, or other closure invariant can persist. The return count N and its correlations may simultaneously supply additional comparators. Persistent identity and accumulating history are then distinct components of one path structure that must be derived separately.

Refinement is not automatically strict when every additional return is completely aliased and neither N nor a new correlation is operational. The ΦBSU no-alias rule is therefore a necessary gate, not rhetorical decoration.

5.4 Three-level dictionary correction

Level	ΦBSU term	Exact object	Information status
I_0	Absolute 0D identity seed	Comparator-free initial object with only the empty identity path modulo gauge	$\mathbb{C}1$, zero bits
Z_Φ	Identity-channel zero-state asymptote	Additive 1R-2R-3T closure-capacity structure	Already a nontrivial premeasurement reservoir
M_Φ	Measurement or curvature event	A local closed comparator whose active conditions are jointly satisfied	Realized distinction and readout

Vacuum Structure states explicitly that, at the zero-state asymptote, the structure is present as distinguishable closure capacity. The $\varepsilon_\emptyset$ zero-state must therefore not be renamed a zero-bit initial algebra. Identity before entropy concerns the I_0 boundary, which precedes Z_Φ.

5.5 Falsifiers of the bridge

The bridge fails if any of the following holds:

- the 0D boundary carries a nontrivial gauge-invariant winding, seam, or holonomy operator;
- a defect or multivalued α produces a topological sector with zero separative support;
- $\mathrm{Path}_\Phi(D_0)/\mathrm{gauge}$ is not the trivial one-object groupoid;
- a later physical quotient identifies an earlier holonomy comparator without an explicit erosion mechanism;
- A_{id} fails to define coherent transport on the Path_Φ groupoid;
- the construction relies only on $\exp(i2\pi n)=1$ and no lifted winding or seam-sensitive readout can be derived;
- the zero-state asymptote's closure capacity is again counted as zero.

6. Theorem C: growth of accessible records and the status of the record arrow

Let $R^{acc}(\tau)$ be the algebra of records physically readable at τ. An additional condition is now required.

R2. Record persistence or recoverability

There exists a faithful channel from the later physical state that preserves or recovers the earlier readout. In the Heisenberg picture this may be represented as an injection

$$\iota_{21}:R^{acc}(\tau_1)\hookrightarrow R^{acc}(\tau_2).$$

R2 is the operational form of an arrow. Decoherence, environmental redundancy, a stable topological record, or another concrete dynamics may realize it. H0-H5 do not derive it.

Theorem C. Conditional monotonicity of record distinguishability

Assume R2, write $R_i=R^{acc}(\tau_i)$, and let ω and φ be normal states on R_2. Restrict the same hypothesis pair to R_1 through ι_{21}. Araki relative entropy obeys

$$S\_Ar(\omega|R_1 \parallel \varphi|R_1) \le S\_Ar(\omega|R_2 \parallel \varphi|R_2).$$

This formulation applies to type-III von Neumann algebras through Araki's construction. The values lie in the extended real line; $\infty\le\infty$ is not evidence of strict physical growth. Strict growth requires an explicit hypothesis pair for which both values are finite and the inequality is proper. Petz sufficiency characterizes a plateau: for that pair, the later record adds no distinguishing information that cannot be recovered from a sufficient earlier algebra.

The mathematical content of Theorem C is data-processing bookkeeping. The physical work lies in deriving R2.

Why Outcome B does not yet derive R2

The finite positive memory sector in Outcome B is reversible and recurrent. A retarded solution follows retarded initial data, but a finite closed Hamiltonian system does not produce one-way record persistence. A causal dissipative readout requires, for example, a continuous positive bath spectrum, a Schwinger-Keldysh in-in reduction, or an explicitly stable topological record.

The study of R2 must not hide inside the generic word isotony. It is a separate dynamical work package in Section 11.

7. Theorem D: the time-slice no-go in its proper domain

Regime condition QFT-reg

Assume a regular globally hyperbolic spacetime, a well-defined locally covariant functor of gauge-invariant observables, and the time-slice axiom. Let N_1 and N_2 be regular neighborhoods containing Cauchy surfaces, with Cauchy morphisms inducing a $*$-isomorphism

$$\alpha_{21}:A(N_1)\cong A(N_2).$$

Theorem D. Invariance of complete-algebra capacity

In the QFT-reg regime, every distinction structure and capacity invariant under $*$-isomorphism is constant. If the complete algebra of one Cauchy neighborhood is $\mathbb{C}1$, every algebra linked to it by time-slice morphisms is isomorphic to $\mathbb{C}1$. Nontrivial present physics cannot emerge from such a complete initial algebra by unitary time-slice evolution.

The proof is immediate: the isomorphism and its inverse transfer every state and comparator bijectively.

Ultraviolet status

Theorem D is a strong no-go for the semiclassical and local-QFT regime. It is not an established law of the initial boundary. At least the following remain open at the Planck regime:

- the definition of local gauge-invariant observables in gravity;
- regional factorization and edge-mode structure;
- the existence of a fixed Hilbert space or Cauchy algebra;
- global hyperbolicity and extension of the time-slice functor to the ultraviolet boundary.

The positive history-prefix theorem does not assume these structures. It does not reject them as low-energy consistency gates. A genuinely refining ultraviolet branch of ΦBSU must recover QFT-reg approximately on regular domains.

The no-go and the refinement theorem concern different objects. $A(\Sigma)$ may be invariant while $C^{H}(D)$ refines through new correlations inside one global algebraic representation. If ΦBSU instead claims a genuinely enlarging complete state space, it must explicitly select an ultraviolet branch outside QFT-reg.

8. Theorem E: continuation multiplicity and the measure-trace boundary

Quantitative definition of continuation classes

Let $q_1 \in QF_1(D_1)$ be an earlier operational prefix class. Its continuation multiplicity is the size of the fiber of the induced class map:

$$m_{21}(q_1) := |rQ_{21}^{-1}(q_1)|.$$

The value may be infinite; for a finite instrument family it is a directly countable integer. It counts admissible continuation classes distinguished by comparators and assumes no probability measure.

Theorem E. An identity boundary and strict refinement force continuation multiplicity

Assume the well-defined quotients of Theorem A together with H2-H3. Refinement relative to the pulled-back earlier partition is strict if and only if

$$\text{there exists } q_1 \in QF_1(D_1) \text{ such that } m_{21}(q_1) \geq 2.$$

If H4 gives $C_0^{+} = \mathbb{C}1$, let $Q_0 = \{*\}$ denote its single operational class. Although D_0 is a filter limit rather than an element of the domain poset, the compatible restrictions in the projective-limit construction preceding Theorem B induce a boundary class map

$$rQ_{D\to 0} : QF(D) \to Q_0,$$
$$m_{D\to 0}(*) := |rQ_{D\to 0}^{-1}(*)|.$$

If $|QF(D)| \geq 2$, then $m_{D\to 0}(*) \geq 2$.

Proof. H2 makes rQ_{21} surjective. A proper refinement means exactly that at least one of its fibers contains two or more later comparator classes. The root corollary follows because the induced boundary map has the one-point target Q_0. ■

For finite instrument families at intermediate domains, continuation multiplicity is instrument-relative: $m_{21} \geq 2$ may reflect either multiple admissible continuations of one physical prefix or distinct physical prefixes that F_1 failed to distinguish and F_2 later resolves. The root corollary is stronger because H4 concerns the complete boundary comparator family. Its single class is not an artifact of limited instrument resolution, so a later distinction entails at least two admissible physical-history classes in the induced boundary fiber.

The theorem counts admissible continuation classes relative to the stated instruments; its complete-boundary corollary is independent of an intermediate instrument choice.

Probability measure as a separate structure

Multiplicity does not determine a continuation measure. If the theory supplies domain measures μD, they must at minimum be prefix-consistent:

$$(rQ_{21})_* \mu D_2 = \mu D_1.$$

Weights of continuation classes are described by conditional measures $\mu D_2(\cdot|q_1)$. On each fiber $rQ_{21}^{-1}(q_1)$, let $\mu_{0,D_2}(\cdot|q_1)$ be a reference conditional measure and define the fiberwise Radon-Nikodym derivative

$$d\mu_{\kappa,D_2}(\cdot|q_1)/d\mu_{0,D_2}(\cdot|q_1) = w_\kappa/Z_\kappa(q_1),$$
$$Z_\kappa(q_1)=\int_{rQ_{21}^{-1}(q_1)} w_\kappa \, d\mu_{0,D_2}(\cdot|q_1).$$

With the earlier marginal fixed, this fiberwise normalization preserves prefix consistency. A proposed deformation is physical only if it also supplies an operational comparator, a predicted measure trace, and consistency with causality and with the observed statistics in its intended regime.

Scenario class	Quantitative status in this memorandum	Consequence
Measure-neutral actualization interpretation	Changes neither μD nor any comparator statistic	Interpretation, not an additional physical claim of this memorandum
Global selection or final condition	May remove earlier continuation classes	Test first at the H2 surjectivity gate
Explicit measure deformation	Supplies w_κ, Z_κ, and an observable deviation	Physical candidate if projective consistency and empirical bounds are satisfied
Complexity-localized feedback, including a possible conscious-choice interpretation	Proposed coherence-positioning filter that could weight paths favorable to developmental variation; no invariant κ, w_κ, or readout is yet known	Open scenario among others; without a measure trace it has the same methodological status here as a measure-neutral many-worlds scenario

The scope is exact: this memorandum studies the quantitative fate of physical information - partitions, multiplicities, measures, and capacities. A continuation-actualization mechanism is an object of the memorandum only through its measure trace.

9. The holographic route remains auxiliary

Bousso's covariant entropy bound

$$S(L)\leq A(B)/(4\ell\_P^2)$$

bounds the entropy flux of a specified state on a suitable light-sheet. By itself it neither bounds the number of pure alternatives nor yields an ℂ1 boundary. Deriving identity from area would require the stronger capacity assumption

$$\ln N(D)\leq A(D)/(4\ell\_P^2),$$

together with an $A\to 0$ limit, permission to form an equal mixture under the same geometry, control of backreaction, and exclusion of zero-area topological sectors. These are species-bound and ultraviolet issues.

The present identity derivation does not require holography. A holographic capacity bound remains an independent corroborating route if it is later derived from the ΦBSU closure structure.

10. Premise and domain-of-validity audit

Object	Exact claim status	Limitation	Open physical work
Theorem A	Analytic pulled-back refinement	Not an independent information-production law	Derive H1-H3 and H5 from ΦBSU dynamics
Carrier types	$\equiv_{F_2,D_2}$ is compared with the relation $\equiv_{21}F_1$ pulled back to $H(D_2)$	Partitions on different sets are not directly compared	Test gauge and comparator descent before forming quotient maps
Accessible records	Araki DPI under R2	R2 is an operational arrow condition	Derive recoverability or provide an error bound δ
Type III	The primary result is the order of finite-instrument partitions	$\infty \leq \infty$ does not demonstrate growth	Choose an instrument family or justified split approximation
Apical identity	Ultraviolet target statement under H4	Verch's interior-point result does not cover an ideal boundary	Derive the closure-cost lemma and audit a residual sector Z_0
QFT-reg no-go	Conditional semiclassical theorem	Not extrapolated to the Planck boundary	Recover time-slice structure in the regular regime
Holonomy bridge	η_D and W_Φ define the target structure	The trivial initial groupoid is still a premise; global dα carries no nonzero winding	Derive Path_Φ(D_0)/gauge={ε}, local lifts, and a faithful readout; exp(i2πn)=1 is insufficient
Theorem E	Quantitative multiplicity of continuation classes	Not a probability law	Derive a measure only through the separate G7 gate

11. ΦBSU research program

The program is not an invitation to collect further entropy analogies. Its first task is to derive or defeat three links: the comparator groupoid, apical trivialization, and non-aliasing history extension. Only after those links are established does it investigate the accessible-record arrow. Continuation selection is included only if it leaves a quantitative support or measure trace.

Critical path CP0-CP6 and transverse Gate G7

CP0. Freeze the objects

Produce one machine-readable dictionary in which I_0, Z_Φ, and M_Φ are distinct states. Record C^H, R^{acc}, and A(Σ) as different objects. Every later information measure must be indexed by its carrier.

Gate G0: no closure-capacity term belonging to the zero-state asymptote may occur in the ℂ1 boundary.

CP1. Construct the ΦBSU comparator groupoid

Construct Path_Φ(D) directly from the $H^A(x,k,\Gamma)$ history field of the centribuoyancy equation and from the null threading of Part I. Define:

- the objects and admissible null-history morphisms;
- composition, object-indexed identity morphisms, and inverses, distinguishing formal path inversion from dynamical time reversal;
- the gauge action on phase, orientation, and closure labels;
- the lifted winding, seam or P_+ character, and possible flat holonomy of a closed comparator;
- the functor induced by domain inclusion.

Gate G1: for every comparison event x, ε_x is its unique identity morphism; composition and inverses satisfy the groupoid laws; the lifted invariant respects composition; and every claimed observable is gauge-invariant. The number exp(i2πn)=1 alone does not pass the gate.

CP2. Derive prefix extension, quotient descent, and no aliasing

Derive the raw restriction $\dot{r}_{21}$ from the transport law of partial histories, not from AQFT isotony. Verify the raw coherence law

$$\dot{r}_{31}=\dot{r}_{21}\circ\dot{r}_{32}.$$

Test gauge descent and comparator descent separately:

$$x\sim_{g,D_2}y \Rightarrow \dot{r}_{21}(x)\sim_{g,D_1}\dot{r}_{21}(y),$$
$$h\equiv_{F_2,D_2}h' \Rightarrow r_{21}(h)\equiv_{F_1,D_1}r_{21}(h').$$

Only after these tests may r_{21} and rQ_{21} be defined. Then test coherence, surjectivity, and injectivity of r_{21}^*. If some prefixes terminate, add a physically interpreted terminal state only when it is an admissible object of the theory. Classify every failure of descent or surjectivity: final-state constraint, annihilation of history, topology change, retroactive change of the gauge quotient, or representational infidelity.

In a linearized truncation, use the explicit system

$$H_{n+1}=U_nH_n+B_ns_n.$$

Construct the block history map from past sources to the present H_n and compute its kernel. A nontrivial kernel is a direct alias witness. The remedies are an append-only record variable, demonstrated fidelity of Γ and N_Γ labels, or an explicit restriction of the theorem to an external cumulative register.

For every new ΦBSU domain, exhibit a comparator g that distinguishes two continuations with the same earlier prefix. Prime-cycle or zero-product language alone is insufficient; independence must be demonstrated in the comparator algebra.

Compute the multiplicity

$$m_{21}(q_1)=|rQ_{21}^{-1}(q_1)|$$

for every finite test fiber. Strict refinement requires at least one value $m_{21}\geq 2$.

Gate G2: the raw restriction descends to both gauge and comparator quotients, the induced prefix maps are coherent, rQ_{21} is surjective, and r_{21}^* is injective. At least one physical update has $m_{21}\geq 2$; an alias alone does not pass.

CP3. Prove the apical closure-cost lemma

Replace the Verch extrapolation with the ΦBSU-native lemma:

> **Every nonconstant gauge-invariant ΦBSU comparator requires a nontrivial closed null history and positive separative support.**

Derive the support requirement from A_id transport, holonomic returns, and closure correlations. Treat defects, multivalued α, zero-area loops, edge modes, and global charges separately.

Gate G3A: if all residual sectors are excluded, $W\_\Phi(D_0)=\mathbb{C}1$.

Gate G3B: if a residual sector Z_0 remains, publish the correct result $W\_\Phi(D_0)=Z_0$ and do not rename it identity.

CP4. Persistent structural identity and growing return history

Define $I(h_N)$ as a sector projection, compensated winding class, seam class, or closure class, and $E:h_N\mapsto h_{N+1}$ as a history extension. Raw winding is additive and is not automatically preserved. Prove separately the conditions under which I(Eh)=I(h) while N_Γ and off-diagonal correlations increase. If P_s projects onto a structural-identity sector s, test [L,P_s]=0 away from explicit defects and open-closed conversion. Separate:

- a return that adds only redundancy, so the partition does not refine;
- a return that opens a new comparator, so H5 holds.

Then attempt to derive a genuinely ΦBSU-local history-production law rather than restating data processing:

$$J_s^\mu(x) = \int_{\Gamma:s(\Gamma)=s} k^\mu H(x,k,\Gamma)\,d\mu(k,\Gamma),$$

$$D_\mu J_s^\mu=\sigma_s^{cl},\quad \sigma_s^{cl}\geq 0.$$

If such a law holds,

$$N_s(\Sigma_2)-N_s(\Sigma_1)=\int_V \sigma_s^{cl}\, dV\geq 0.$$

Strict information growth still requires $\sigma_s^{cl}>0$ and proof that the produced record is not a function of the old comparator algebra. The sign $\sigma_s^{cl}\geq 0$ must be derived from a positive closure kernel; it may not be inserted by notation.

Also derive the candidate measurement morphism

$$M_\Gamma:(A_{id},A_{geom},P_+)\mapsto E_\Gamma = P_+ \mathrm{Hol}_\Gamma(A_{geom}+A_{id})P_+$$

and identify the lifted or seam-sensitive component by which E_Γ distinguishes histories. This is the concrete interaction-scale bridge in Part I from identity transport to readout.

Gate G4: return by return receives an explicit operator, invariant, multiplicity, and novelty test.

CP5. Accessible-record dynamics

Derive R2 or falsify it in the LTMC/open-closed memory sector. A finite oscillator set is insufficient. Test at least:

1. a continuous positive bath spectrum and an in-in influence functional;
2. a stable topological record sector;
3. growth of environmental redundancy;
4. Petz recovery error and conditional mutual information;
5. record-erasure and recurrence counterexamples.

If exact recovery fails, construct a decoder $\mathcal{D}$ and report a uniform error bound δ. For example, on the selected code space the trace-distance distinction should satisfy

$$\Delta(E\rho,E\sigma)\geq\Delta(\rho,\sigma)-\delta.$$

This gives an honest effective result without overstating exact isotony.

Gate G5: supply either a dynamic quantitative record-persistence bound or mark R2 as a separate arrow postulate. A retarded Green function alone does not pass.

CP6. Regime bridge and observational test

Show how a genuinely refining history-prefix branch recovers QFT-reg time-slice/unitary behavior on regular domains without contradiction. Only then attach the interaction-scale observation operator of Part II to an accessible-record or memory question, not to the proof of the identity boundary.

Candidate observables include the frequency dependence of a dynamical memory-response channel, quadrature information, and a novelty measure for return correlations. Static single-time data do not identify an arrow.

Gate G6: provide at least one synthetic recovery test, one alias negative control, one record-erasure control, and one regular-domain unitarity test.

G7. Measure-trace boundary gate

G7 is not a premise of Theorems A, B, or E. It opens only if weights are proposed for continuation classes. The proposal must provide an explicit positive normalized measure or weighting functional, prove gauge invariance and prefix compatibility $(rQ_{21})_*\mu D_2=\mu D_1$, and identify an operational measure trace or a known statistical bound. Without these, the proposal is classified as an interpretive possibility and is not used as a ΦBSU premise, result, or prediction. If a global support constraint violates H2, the analysis returns to the global-constraint branch.

Paradigm assumptions bypassed by the positive branch

Theorems A, B, and E do not assume:

- a fixed background manifold or an interior point p_0;
- a complete Cauchy algebra at the Planck boundary;
- global unitarity or a constant ultraviolet Hilbert-space dimension;
- factorization into local Hilbert spaces;
- finite capacity of a type-III AQFT algebra;
- extrapolation of Bousso or QFC to $A\to 0$;
- one invariant microscopic front speed;
- thermodynamic record persistence;
- a probability law or an actualization mechanism for continuation fibers.

They use the smaller history-logical core H0-H5. This does not license arbitrary ultraviolet physics. Every bypassed assumption returns as its own low-energy, causal, positivity, measure, or observational gate.

Mandatory outputs

1. comparator_groupoid.md: generators, relations, gauge quotient, and holonomy functor.
2. prefix_functor.py: quotient-descent, coherence, surjectivity, pullback-injection, and multiplicity tests on a finite truncation.
3. apex_sector_audit.md: trivialization or a residual center Z_0.
4. novelty_operator.md: the non-aliasing event condition and at least one computed example.
5. record_channel.py: recovery fidelity, Petz gap, or an explicit arrow postulate.
6. regime_bridge.md: correspondence between the ultraviolet history branch and regular-domain LTMC/QFT-reg.
7. continuation_measure.md: either a normalized, testable weighting functional or an explicit statement that no measure claim is made.
8. falsification_matrix.csv: a countermodel and observable consequence for every gate.

Stopping rules

- If quotient descent fails in CP2, no single physical prefix functor exists and universal history refinement is withdrawn.
- If surjectivity fails in CP2, universal history refinement is withdrawn and the theory is classified as a global-constraint branch.
- If CP3 leaves a residual center, the identity boundary is replaced by a sectorial initial algebra Z_0.
- If all returns in CP4 are aliases, history grows denotes only an order parameter, not production of information.
- If CP5 does not derive R2, accessible-record growth is not presented as a theorem.

- If G7 supplies no measurable weighting functional, no continuation-selection narrative appears in the physical conclusions.
- If CP6 fails to recover low-energy unitarity and observed causality, the ultraviolet branch is rejected regardless of its information-logical elegance.

Outcome classes

- **Outcome A - strong ΦBSU result:** prefix extendibility, quotient descent, a faithful comparator functor, a non-aliasing event, apical trivialization, and the regular-domain bridge are derived. History distinctions refine along every faithful event foliation and begin at $\mathbb{C}1$. A continuation measure is not required for this result.
- **Outcome B - effective recovery result:** injection or record persistence holds only in a bounded time, energy, or resolution domain with error δ. Publish approximate monotonicity and the decoder bound, not an exact ultraviolet theorem.
- **Outcome C - structured boundary:** a residual center, zero-support holonomy, or another boundary sector remains. The initial algebra is $Z_0 \neq \mathbb{C}1$, and identity before entropy survives only as the categorical statement about the empty history.

12. Serious routes to the opposite result

Failed condition	Opposite conclusion	Serious physical realization
H0 extensionality	Identity is not the beginning because indistinguishable primitive labels are counted separately	Haecceity, hidden sector, or ontic label with no possible comparator
H1 common prefix order	Across all foliations has no shared meaning	Indefinite causal order, chronology failure, or incompatible event posets
H2 surjective extendibility	A later admissible history space may exclude an earlier prefix	Final-state boundary condition, global consistency selection, topology-changing exclusion
H3 quotient descent	No physical restriction map exists	A later gauge identification has no consistent action on earlier physical classes
H3 faithful readout	The represented distinction reservoir may decrease	Coarse-graining, finite-window truncation, deliberate forgetting
H4 apical trivialization	The boundary is $Z_0 \neq \mathbb{C}1$	Topological holonomy, edge mode, superselection charge, zero-support defect
H5 non-aliasing novelty	Strict refinement disappears	Bijective induced class map rQ_{21} or operational redundancy
All continuation fibers have $m=1$	No strict history distinction can arise from a comparator-free root	A globally unique admissible continuation relative to every prefix quotient
No cosmological initial object	Identity is a logical but not temporal beginning	Bounce, past-eternal, or cyclic cosmology
R2 record persistence	Currently readable record capacity may decrease	Uncomputing, record erasure, recoherence, finite-memory overwrite
QFT-reg time slice	Complete Cauchy-algebra capacity cannot grow	Standard unitary regular-domain QFT
Every later distinction is already comparator-readable in initial data	The physical initial algebra is nontrivial rather than $\mathbb{C}1$	Complete initial-data encoding with a boundary comparator for every later distinction
Time-reversed boundary condition	The record or entropy arrow reverses	Final Past Hypothesis or a two-ended boundary condition

The sharp alternative is quantitative. If a later strict distinction exists relative to a comparator-free root, some continuation fiber must contain at least two comparator classes. If every later difference is already readable by a boundary comparator, the boundary algebra was not $\mathbb{C}1$. If a global solution label exists but is not readable by any boundary comparator, H0 places its alternatives in a later continuation fiber relative to the physical quotient; the memorandum does not silently count that label as initial information.

No qualitative label resolves this fork. ΦBSU must derive the relevant restriction maps, fibers, and comparators, or accept that the zero-bit boundary is only a quotient-level statement.

13. Source-faithful map of ΦBSU claims

Source	Pages or equations	Safe use in this manuscript
ΦBSU Part I	PDF pp. 1, 3-5, 16-17	$A_{id}=-i\Phi^{-1}d\Phi$ with local lifts $A_{id}\vert U_i=d\alpha_i$; global phase data become operational in closed comparators; local pure gauge must be distinguished from winding, seams, and closure readout
ΦBSU Part II	PDF p. 9	Does not yet provide a complete law of cosmological entropy or growing history
Quark-Gluon Interiors	PDF pp. 1-2	Complete sameness is not doubled existence; an event begins with irreducible separation
Vacuum Structure	PDF pp. 2-3	0D identity seed, absence of an external comparator, first self-counterpart distinction, and no-alias domain rule
Vacuum Structure	PDF p. 6	The zero-state asymptote already contains distinguishable closure capacity and is not $\mathbb{C}1$
Centribuoyancy	PDF p. 2, eq. 3	$q\vert U_i=d\alpha_i$ as a local starting point of metric-free identity and phase transport; a nonzero period requires transition or multivaluedness data
Centribuoyancy	PDF p. 5, eqs. 21-23	$H^\wedge(x,k,\Gamma)$ as a causally accumulated null-history state; transport and retarded representation
Centribuoyancy	PDF p. 14, eqs. 76-77	Holonomic return, N_Γ return history, and the statement that structural identity persists
Outcome B	Final report §3 and equations in §6	A finite positive memory sector is reversible; a dissipative record requires a bath or in-in reduction

The project sources do not yet prove H2-H5. They supply a native dictionary and some ingredients. The prefix-extension lemma, the corrected pullback relation, continuation multiplicity, the apical holonomy bridge, and the separation of three reservoirs are the new formal synthesis of this memorandum.

14. References

External primary sources

1 R. Haag and D. Kastler, "An Algebraic Approach to Quantum Field Theory," *Journal of Mathematical Physics* **5**, 848-861 (1964). https://doi.org/10.1063/1.1704187

2 R. Brunetti, K. Fredenhagen, and R. Verch, "The Generally Covariant Locality Principle - A New Paradigm for Local Quantum Field Theory," *Communications in Mathematical Physics* **237**, 31-68 (2003). https://doi.org/10.1007/s00220-003-0815-7

3 R. Verch, "Local Definiteness, Primarity and Quasiequivalence of Quasifree Hadamard Quantum States in Curved Spacetime," *Communications in Mathematical Physics* **160**, 507-536 (1994). https://doi.org/10.1007/BF02173427

4 H. Araki, "Relative Entropy of States of von Neumann Algebras," *Publications of the Research Institute for Mathematical Sciences* **11**, 809-833 (1976). https://doi.org/10.2977/PRIMS/1195191148

5 G. Lindblad, "Completely Positive Maps and Entropy Inequalities," *Communications in Mathematical Physics* **40**, 147-151 (1975). https://doi.org/10.1007/BF01609396

6 D. Petz, "Sufficient Subalgebras and the Relative Entropy of States of a von Neumann Algebra," *Communications in Mathematical Physics* **105**, 123-131 (1986). https://doi.org/10.1007/BF01212345

7 C. J. Fewster, "The Split Property for Quantum Field Theories in Flat and Curved Spacetimes," *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg* **86**, 153-175 (2016). https://arxiv.org/abs/1601.06936

8 R. Geroch, E. H. Kronheimer, and R. Penrose, "Ideal Points in Space-Time," *Proceedings of the Royal Society A* **327**, 545-567 (1972). https://doi.org/10.1098/rspa.1972.0062

9 R. Bousso, "A Covariant Entropy Conjecture," *Journal of High Energy Physics* **07**, 004 (1999). https://arxiv.org/abs/hep-th/9905177

10 W. Donnelly and L. Freidel, "Local Subsystems in Gauge Theory and Gravity," *Journal of High Energy Physics* **09**, 102 (2016). https://doi.org/10.1007/JHEP09(2016)102

11 S. L. Braunstein and A. K. Pati, "Quantum Information Cannot Be Completely Hidden in Correlations: Implications for the Black-Hole Information Paradox," *Physical Review Letters* **98**, 080502 (2007). https://arxiv.org/abs/gr-qc/0603046

12 J. Yngvason, "The Role of Type III Factors in Quantum Field Theory," *Reports on Mathematical Physics* **55**, 135-147 (2005). https://arxiv.org/abs/math-ph/0411058

Project sources

S1. *A New Reflection-Graded Hypersymmetry on* $M_4 \times K_2$, updated 2026.

S2. *ΦBSU Field Theory, Part I: Completing Relativity*, 1 January 2026.

S3. *A New 4-Dimensional Cosmological Principle and Buoyant Spacetime*, 2024.

S4. *ΦBSU Field Theory, Part II: Completing Information Closure toward Topological Background-Independence*, revised 2026.

S5. *Supplementary Reproducibility Note for the Revised ΦBSU Part II Closure*, 2026.

S6. *Background-Independent Quark-Gluon Interiors with Dark-Sector Outeriors via* 3^2 *Holonomy Nodes*, May 2026.

S7. *Centribuoyancy in a Causal Null-History Field*, August 2026.

S8. *A Background-Independent Vacuum-Structure Prediction for the Measured Fine-Structure Constant and Charged-Lepton Mass-Amplitude Ratios*, 4 August 2026.

S9. *ΦBSU History Closure Outcome B*, research package, 8 August 2026.

Final claim level

The project sources support comparator-closure language, a 0D identity seed, a non-aliasing event principle, local $q|U_i=d\alpha_i$ transport, holonomic return, and return-by-return history. They do not yet derive surjectivity of the physical prefix maps, quotient descent, apical-sector trivialization, an accessible-record arrow, or a continuation measure from the complete dynamics.

This memorandum establishes the general order-theoretic information framework and states a ΦBSU-specific morphism candidate with its derivation gate. Its strongest quantitative corollary is continuation multiplicity: strict refinement requires at least two comparator-distinguishable continuation classes in one operational prefix fiber. The remaining dynamical, apical, record, regime, and optional measure claims are explicit falsifiable gates rather than hidden premises.