A Background-Independent Vacuum-Structure Prediction for the Measured Fine-Structure Constant and Charged-Lepton Mass-Amplitude Ratios

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Abstract

Conditional on an explicitly stated quadratic comparator-norm axiom, a finite closure-domain grammar, and the physical bridge hypotheses identified below, this paper develops a background-independent vacuum construction and two empirical applications. Operational equivalence defines what counts as a new event. Three successive premetric closure alphabets have support cardinalities $N_1 = 2^1$, $N_2 = 3^2$, and $N_3 = 5^3$. Transitive full-support closure and positive weight-preserving transfer operators obeying graded closure covariance yield the normalized transfer densities $N_1/N_2 = 2/9$ and $N_2/N_3 = 9/125$. A weighted identity-retention bridge then defines the exact zero-comparison inverse-coupling asymptote

$$\varepsilon_\emptyset = 1 + 2 + 3^2 + 5^3 + \frac{1}{2}\frac{3^2}{5^3} = 137.036,$$

with a full-information endpoint $5^3 = 125$.

Measured values of α^{-1} are treated as apparatus-dependent manifestations of this fixed interval, not as revisions of the fundamental asymptote. We introduce the discrete apparatus grammar

$$g_A = 1 + \frac{\chi_A}{r_A}, \qquad \chi_A \in \{0, 1\},$$

where r_A is the phase-transfer rank and χ_A is the rank of an uncancelled closure mode after the experimental symmetry projections. The Raman–Bloch rubidium readout is assigned $(\chi, r) = (0, 1)$, whereas a fifth-order same-internal-state Bragg readout is assigned $(\chi, r) = (1, 5)$, giving $(g_{\mathrm{Rb}}, g_{\mathrm{Cs}}) = (1, 6/5)$. This is a conditional apparatus-topology conjecture, not a result of standard atom-interferometer theory. Postdictively, the mutually discrepant Rb and Cs recoil determinations triangulate

$$\widehat{\varepsilon_\emptyset} = 6\alpha_{\mathrm{Rb}}^{-1} - 5\alpha_{\mathrm{Cs}}^{-1} = 137.036000006(150),$$

assuming independent quoted uncertainties. A separate canonical quadratic bridge $\delta_0 = (3^2 5^3)^{-2}$ gives the absolute double postdiction 137.0359992098765 for the Rb protocol and 137.0359990518519 for the fifth-order Cs protocol, differing from the published results by 0.35σ and 0.22σ. The triangulated amplitude, the Rb-only prospective calibration, and the canonical amplitude are kept explicitly distinct.

Four prospective tests are registered: a same-apparatus Bragg-order sweep linear in $1/n$; a two-species Raman/Bragg cross-over test including the maximal $n = 1$ contrast; persistence of the fifth-order Bragg class after improved conventional systematics; and a Penning-trap $g-2$ channel assigned to the closure-free class $\chi = 0$, conditional on the adopted high-order QED and other Standard-Model inputs. A printed withdrawal criterion requires one universal closure operator to derive both $g_{\mathrm{Raman}} = 1$ and $g_{\mathrm{Bragg}}(n) = 1 + 1/n$ without apparatus-specific continuous constants.

For charged leptons, a C_3 mass-amplitude spectrum gives $K = 1/3 + a^2/6$. The Koide value 2/3 follows conditionally from the same quadratic comparator-norm axiom, which fixes $a = \sqrt{2}$.

Graded closure covariance supplies the transfer weight 2/9; its oriented radian embedding $\vartheta_K = -2/9$ and the (τ, μ, e) closure-order assignment remain physical hypotheses. The resulting m_τ/m_μ value is a successful zeroth-order postdiction, consistent with PDG 2026 at 0.42σ and prospectively testable by improved tau-mass measurements. The precisely known m_μ/m_e ratio differs by about 452σ; no electron-tail or common-node correction is fitted here.

Keywords: background independence; vacuum structure; fine-structure constant; atom recoil; apparatus topology; graded closure covariance; Koide relation; charged-lepton mass amplitudes.

1 Scope, claim status, and order of construction

This article is a physics construction supported by a finite information grammar; it is not presented as a general theorem of information theory. The logical order is

$$\begin{aligned}\text{operational distinction} &\longrightarrow \text{premetric closure domains}\\ &\longrightarrow \text{closure-covariant transfer}\\ &\longrightarrow \text{physical readout map}\\ &\longrightarrow \text{empirical consequence.}\end{aligned}$$

The first three steps define the internal structure. The last two are physical bridges and must be assessed experimentally.

Four statuses are distinguished throughout:

(i) an *internal result* follows from the stated finite grammar;

(ii) a *physical bridge* identifies an internal functional with an observable;

(iii) a *postdiction* uses data already available when the bridge was formulated;

(iv) a *registered prospective prediction* is frozen before the relevant new data.

The Rb–Cs relation and its triangulation are postdictive. The charged-lepton ratios are also postdictive in the present paper; improved tau-mass measurements provide their prospective continuation. The four apparatus tests in Sec. 6 are the registered prospective apparatus content.

The wider ΦBSU programme uses a global phase $\Phi = e^{i\alpha}$, an identity connection $A_{\mathrm{id}} = d\alpha$, and a separate local curvature channel $F_{\mathrm{geom}} = dA_{\mathrm{geom}}$ [18]. Only the minimum structure required for the present vacuum and lepton applications is used below.

Reader's guide: from distinguishability to the vacuum number

The construction is easiest to follow if one temporarily postpones the formal operators and asks a simpler question: how can distinct information structures be aligned when no space, ruler, coordinate system, or common interaction manifold has yet been assumed?

The word *co-localized* is used here in this pre-geometrical sense. It does not mean that several objects have been placed at the same coordinate point. It means that successive information domains refer to one underlying event and must remain mutually identifiable without an external reference frame. Before the translational domain $3T$ has opened, the model assumes no operational spatial metric, no operationally non-zero cross-domain inner products, and no shared manifold on which relative domain directions could already be measured. The quadratic comparator norm introduced in Sec. 2.2 is an internal rule relating a state to its self-counterpart; it is not a spatial metric between domains and does not supply the missing pre-$3T$ geometry. Common alignment must therefore be supplied by internal symmetry: no position or direction may be preferred, and every configuration needed for a domain's own closure must be available on equal terms.

The starting point is one undivided identity record, denoted schematically as a $0D$ seed. Merely copying this record would not create a second event, because two completely indistinguishable records add no separative information. The first genuine distinction must therefore be two-sided: the state is compared with an antipodal counterpart. This opens the first rotational domain, $1R$, with the minimal two-position closure alphabet

$$N_1 = 2^1.$$

The symbols N_1, N_2, and N_3 are the support cardinalities used throughout the formal sections. The exponent records the dimension of the domain; the base records the minimal non-aliasing cycle used in that domain.

A further independent distinction cannot be obtained by repeating the same binary split. Repetition would refine the existing $1R$ distinction rather than create a new kind of neighborhood. The model therefore uses the next cycle that is not divisible by the earlier one, namely 3. Applied symmetrically in a two-dimensional internal domain, it gives the phase-surface alphabet

$$N_2 = 3^2 = 9.$$

All nine positions are admitted as the support of the domain. If only a proper subset were available before any geometry existed, the missing positions would already select preferred directions or locations without a relation that could justify the preference. In this sense full support is not an added thermodynamic assumption; it is the finite-alphabet expression of background-independent homogeneity and isotropy.

The same rule is used once more. The next minimal cycle that does not alias with the earlier cycles 2 and 3 is 5. Its symmetric realization in the first common translational neighborhood domain gives

$$N_3 = 5^3 = 125.$$

At this stage continuous neighborhoods can be represented inside a shared three-direction translational structure. Under the primitive-domain termination hypothesis stated explicitly in Sec. 2.3, later distinctions are represented as directions, fibres, curvature, holonomy, interaction, or further resolution within the common domain rather than by another primitive topological domain. The primary comparator architecture therefore closes at

$$1R + 2R + 3T, \qquad 1 + 2 + 3 = 6.$$

The six here are information-comparator components, not six directly observed spacetime coordinates.

Domain	Minimal cycle	Capacity	Information-theoretic reading
$1R$	2	2^1	First antipodal distinction: one state and its self-counterpart.
$2R$	3	3^2	Closed internal phase surface with full symmetric support.
$3T$	5	5^3	First common translational neighborhood alphabet supporting continuous separation.

The powers 2^1, 3^2, and 5^3 are therefore configuration capacities, not particle counts. The first non-integer term in the construction has a different role:

$$\frac{3^2}{2^1 5^3} = \frac{9}{250} = 0.036.$$

It does not add a fractional state to the alphabets. It is a normalized *surface-to-bulk retention weight.* The numerator 3^2 is the internal phase-surface multiplicity. The factor 5^3 is the translational neighborhood capacity into which that phase information is distributed. The factor 2^1 accounts for antipodal identification in a one-branch operational reading. Thus 0.036 records how much of the internal phase-surface identity remains distinguishable per antipodally normalized translational bulk capacity.

An intuitive, but deliberately limited, analogy is a dimensionless surface-to-bulk tension: the internal phase surface must remain identifiable while it is spread over a larger neighborhood alphabet. This is not a mechanical force or an energy density; in the present paper it denotes only a normalized retention relation between finite information domains.

The vacuum bookkeeping number is then read term by term:

$$\varepsilon_{\emptyset} = 1 + 2 + 3^2 + 5^3 + \frac{3^2}{2^1 5^3} = 137.036.$$

Here 1 denotes undivided identity, 2 the first antipodal distinction, 3^2 the closed internal phase surface, 5^3 the common translational granulation, and 0.036 the projected retention of the internal surface in that translational domain. The equation is a finite identity-retention functional. Its identification with an inverse electromagnetic coupling is a separate physical bridge hypothesis, and apparatus-dependent precision readouts are a still later layer.

A compact reading order for the rest of the paper is therefore:

1. the finite powers describe full-support information alphabets;
2. the ratios between alphabets describe normalized transfer or retention;
3. their additive identity readout gives the proposed vacuum asymptote;
4. only after that internal construction is fixed are bridges to measured α and charged-lepton amplitudes introduced.

2 Information-separation foundation

2.1 Operational equivalence and zero-state levels

Definition 2.1 (Operational equivalence)**.** *Let $\mathcal{S}$ be candidate descriptions and $\mathcal{M}$ a permitted comparator family. Define*

$$s \sim_{\mathcal{M}} t \quad \iff \quad p(y \mid s, M) = p(y \mid t, M)$$

for every $M \in \mathcal{M}$ and every admissible outcome y. A physical state is an equivalence class $[s]_{\mathcal{M}}$.

Complete sameness is not duplicated physical existence: duplicating a label without creating a new comparator-equivalence class adds no separative information. A new event begins only when a new operational distinction is formed.

We separate three levels:

- Z_0: no realized distinction or causal comparison;
- Z_1: finite closure alphabets are structurally admitted, but no local comparator event is realized;
- Z_2: alphabet components meet in an operational comparator and produce a measured outcome.

Thus the symbol $\emptyset$ in $\varepsilon_{\emptyset}$ denotes zero realized causal comparison, not a zero-cardinality alphabet.

2.2 Quadratic comparator norm and the $\sqrt{2}$ update

The first self-counterpart update requires an additional assumption that must be stated rather than hidden inside the word "norm."

Structural axiom 2.2 (Quadratic comparator norm)**.** *The premetric comparator space carries a positive quadratic form Q. A primitive component x and its background-independent counterpart $x^{\perp}$ are equal-norm and Q-orthogonal:*

$$Q(x, x^{\perp}) = 0, \qquad Q(x, x) = Q(x^{\perp}, x^{\perp}).$$

It follows that

$$Q(x + x^{\perp}, x + x^{\perp}) = 2Q(x, x),$$

so the induced update factor is

$$\boxed{\Lambda = \sqrt{2}.}$$

The factor is therefore conditional on background-independent counterpart formation *and* the quadratic comparator-norm axiom. The same equal-norm geometry will reappear in the charged-lepton amplitude sector.

2.3 Successive domains and finite alphabets

The first operational distinction is antipodal and uses the primitive cycle 2. A genuinely new domain cannot reuse a period divisible by an earlier primitive cycle; otherwise it is a refinement of an existing distinction rather than a new non-aliasing closure class. We use the minimal sequence

$$p_1 = 2, \qquad p_2 = 3, \qquad p_3 = 5,$$

and the domain alphabets

$$\Omega_d = (\mathbb{Z}_{p_d})^d, \qquad N_d = |\Omega_d| = p_d^d.$$

Hence

$$N_1 = 2^1 = 2, \qquad N_2 = 3^2 = 9, \qquad N_3 = 5^3 = 125.$$

The pre-operational architecture is

$$R_{\text{int}} := R^{(1)} \times R^{(2)}, \qquad \dim R_{\text{int}} = 3,$$

$$\mathcal{B}_6 = R_{\text{int}} \oplus T^{(3)}, \qquad \dim \mathcal{B}_6 = 1 + 2 + 3 = 6.$$

This is a six-component comparator architecture, not a claim that observed spacetime has six directly measured dimensions. Termination after the common three-direction translational domain is a structural hypothesis: later distinctions are represented as curvature, fibres, holonomy, interaction, or refinement within the common domain.

The cardinality N_d is distinct from Shannon entropy. Its Hartley information is $H_0(D_d) = \ln N_d$ [1]; a distribution P_d on Ω_d has Shannon entropy $H(P_d) \leq \ln N_d$ [2].

3 Full-support closure and graded closure covariance

Let a finite symmetry group G_d act transitively on Ω_d.

Lemma 3.1 (Full-support closure)**.** *If a nonempty support $A \subseteq \Omega_d$ is invariant under every element of a transitive action G_d, then $A = \Omega_d$.*

The lemma is elementary; the physical content resides in the structural requirement that a premetric background-independent closure be invariant under the full internal symmetry action rather than occupy a preferred proper subset.

Let $V_d = \mathbb{R}^{\Omega_d}$ and let C_d be the Reynolds closure projector. A positive transfer map

$$\mathsf{U}_{d,d+1} : V_d \to V_{d+1}$$

is required to preserve total weight and commute with closure:

$$\langle \mathbf{1}_{d+1}, \mathsf{U}_{d,d+1} f \rangle = \langle \mathbf{1}_d, f \rangle,$$

$$\mathsf{C}_{d+1}\mathsf{U}_{d,d+1} = \mathsf{U}_{d,d+1}\mathsf{C}_d.$$

Proposition 3.2 (Graded closure transfer)**.** *For transitive closure orbits,*

$$\mathsf{U}_{d,d+1}\mathbf{1}_d = \frac{N_d}{N_{d+1}}\mathbf{1}_{d+1}.$$

Proof. Closure covariance maps the one-dimensional invariant subspace of V_d into the invariant subspace of V_{d+1}, so $\mathsf{U}_{d,d+1}\mathbf{1}_d = c\mathbf{1}_{d+1}$. Weight preservation gives $cN_{d+1} = N_d$. □

Consequently,

$$\boxed{s_{12} = \frac{N_1}{N_2} = \frac{2}{9},} \qquad \boxed{s_{23} = \frac{N_2}{N_3} = \frac{9}{125}.}$$

A one-sheet antipodal projector gives

$$p_{23} = \frac{1}{2}s_{23} = \frac{1}{2}\frac{9}{125} = 0.036.$$

This quantity is a projected retention weight, not a fractional number of states.

4 Vacuum identity-retention asymptote

In the identity channel, closure grades are retained additively. The weighted identity functional is

$$R_{\mathrm{id}} = 1 + N_1 + N_2 + N_3 + p_{23}.$$

Physical bridge hypothesis 4.1 (Vacuum readout bridge)**.** *The zero-comparison inverse-coupling asymptote is the weighted identity functional:*

$$\boxed{\varepsilon_{\emptyset} = R_{\mathrm{id}} = 1 + 2 + 3^2 + 5^3 + \frac{1}{2}\frac{3^2}{5^3} = 137.036.}$$

The full-information endpoint is

$$\varepsilon_E = 5^3 = 125.$$

The proposed vacuum permeability interval is therefore

$$125 \le \varepsilon_\Phi(\mathcal{I}) \le 137.036, \qquad \alpha_\Phi(\mathcal{I}) = \varepsilon_\Phi(\mathcal{I})^{-1},$$

$$\boxed{\frac{1}{137.036} \le \alpha_\Phi(\mathcal{I}) \le \frac{1}{125}.}$$

Here $\mathcal{I} \in [0,1]$ is only a normalized realization/saturation coordinate: $\mathcal{I} = 0$ labels the Z_1 zero-comparison endpoint and $\mathcal{I} = 1$ the full-information endpoint. No interpolation law for $\varepsilon_\Phi(\mathcal{I})$ is assumed in this article. The endpoints are structural readouts. The map from the zero-state endpoint to a specific precision measurement is treated separately below and is allowed to depend on the operational measurement architecture.

5 Apparatus-dependent fine-structure readouts

5.1 A discrete apparatus grammar

Let

$$a_A := \alpha_A^{-1}$$

denote the value inferred by apparatus class A. We write the leading manifestation map as

$$\boxed{a_A = \varepsilon_\emptyset - g_A\delta + O(\delta^2),}$$

where δ is one universal small manifestation amplitude and g_A is a discrete apparatus factor.

Apparatus-topology conjecture 5.1 (Apparatus-topology grammar)**.** *After all explicit symmetry reversals and standard analysis projections, let r_A be the rank of the independent phase-transfer intervals and $\chi_A \in \{0, 1\}$ the rank of an uncancelled closure/boundary mode. Then*

$$\boxed{g_A = 1 + \frac{\chi_A}{r_A}.}$$

The grammar is deliberately narrow. No apparatus-specific continuous parameter is allowed.

5.2 Rb Raman–Bloch and Cs Bragg–Bloch assignments

The 2020 Rb determination measures a recoil-velocity change with a Raman Ramsey–Bordé readout and combines Raman-direction and Bloch-acceleration reversals [4]. The two arms carry different internal-state labels. The present conjecture assigns the post-projection topology

$$(\chi_{\mathrm{Rb}}, r_{\mathrm{Rb}}) = (0, 1), \qquad \boxed{g_{\mathrm{Rb}} = 1.}$$

The 2018 Cs determination uses same-internal-state Bragg diffraction, Bloch oscillations, and simultaneous conjugate Ramsey–Bordé interferometers [5]. Fifth-order Bragg diffraction is intrinsically multiport and can support parasitic interferometers and diffraction phases [6, 7]. The present conjecture assigns one uncancelled closure mode to the five phase-transfer intervals:

$$(\chi_{\mathrm{Cs}}, r_{\mathrm{Cs}}) = (1, 5), \qquad \boxed{g_{\mathrm{Cs}} = 1 + \frac{1}{5} = \frac{6}{5}.}$$

This is not a result of standard Bragg theory. It is the discrete ΦBSU apparatus-topology hypothesis to be derived or falsified by a full transfer-matrix calculation.

5.3 Why Fisher information alone is insufficient

Fisher information controls local distinguishability and estimator variance; it does not by itself shift the mean. A bias can arise if a real small channel is present but omitted from the fitted model. Let θ be the recoil parameter and η the proposed closure channel. If the true distribution is $p_A(y \mid \theta, \eta)$ but the fitted model uses $p_A(y \mid \theta, 0)$, the first-order pseudo-true displacement has the generic form

$$\boxed{\delta\theta_A = I_{\theta\theta,A}^{-1} I_{\theta\eta,A}\, \eta + O(\eta^2),}$$

with

$$I_{\theta\theta,A} = \mathbb{E}[(\partial_\theta \ln p_A)^2], \qquad I_{\theta\eta,A} = \mathbb{E}[(\partial_\theta \ln p_A)(\partial_\eta \ln p_A)].$$

This is standard misspecified-estimator logic [3]; the model-specific content is the existence and topology of the omitted channel, not the statistical mechanism itself.

5.4 Postdictive Rb–Cs relation and triangulation

The published determinations are

$$a_{\mathrm{R}} = 137.035999206(11), \qquad a_{\mathrm{C}} = 137.035999046(27).$$

Relative to the proposed fixed asymptote,

$$\Delta_{\mathrm{R}} = \varepsilon_{\emptyset} - a_{\mathrm{R}} = 7.94 \times 10^{-7},$$

$$\Delta_{\mathrm{C}} = \varepsilon_{\emptyset} - a_{\mathrm{C}} = 9.54 \times 10^{-7}.$$

Their ratio is

$$\boxed{\frac{\Delta_{\mathrm{C}}}{\Delta_{\mathrm{R}}} = 1.2015 \pm 0.0379,}$$

consistent with 6/5.

Assuming

$$a_{\mathrm{R}} = \varepsilon_{\emptyset} - \delta, \qquad a_{\mathrm{C}} = \varepsilon_{\emptyset} - \frac{6}{5}\delta,$$

the two measurements imply

$$\boxed{\widehat{\delta} = 5(a_{\mathrm{R}} - a_{\mathrm{C}}) = (8.00 \pm 1.46) \times 10^{-7},}$$

$$\boxed{\widehat{\varepsilon_{\emptyset}} = 6a_{\mathrm{R}} - 5a_{\mathrm{C}} = 137.036000006 \pm 0.000000150.}$$

The difference from 137.036 is 0.04 times the quoted propagated uncertainty. This uncertainty assumes the published Rb and Cs errors are independent; common external constants would require a covariance treatment. Since the measurements were known when the 6/5 interpretation was formulated, this is a postdictive triangulation, not a discovery significance.

Three nearby amplitudes occur below in logically different roles and are not identified with one another. The two-data postdictive triangulation estimate is

$$\widehat{\delta} = 8.00 \times 10^{-7}.$$

The frozen Rb-only calibration used in P1–P3 is

$$B_{\mathrm{R}} := \varepsilon_{\emptyset} - a_{\mathrm{R}} = 7.94 \times 10^{-7}.$$

The canonical absolute bridge hypothesis is

$$\delta_0 = 1125^{-2} = 7.901234568 \times 10^{-7}.$$

The Rb-only choice for B_{R} deliberately keeps the Cs datum outside the prospective Bragg-order line calibration. The CODATA 2022 recommended value is a least-squares adjusted combination of several experimental and theory inputs and is therefore not a clean single apparatus-class test point for the present grammar [9].

5.5 Canonical absolute double postdiction

The surface–volume latent alphabet has size

$$N_0 = N_2 N_3 = 9 \cdot 125 = 1125.$$

The following absolute normalization is retained as a separate and more fragile physical bridge.

Physical bridge hypothesis 5.2 (Canonical quadratic manifestation amplitude)**.** *The single-closure manifestation amplitude is*

$$\boxed{\delta_0 = N_0^{-2} = 1125^{-2}.}$$

Together with the apparatus factors, it gives

$$\boxed{a_{\mathrm{Rb}}^{(0)} = 137.036 - \delta_0 = 137.0359992098765,}$$

$$\boxed{a_{\mathrm{Cs}}^{(0)} = 137.036 - \frac{6}{5}\delta_0 = 137.0359990518519.}$$

Table 1: Canonical absolute double postdiction. The quoted deviations use the experimental standard uncertainties. The normalization δ_0 is a physical bridge, not an internal closure theorem.

Protocol	Model readout	Published readout	Deviation
Rb Raman–Bloch	137.0359992098765	137.035999206(11)	0.35σ
Cs fifth-order Bragg–Bloch	137.0359990518519	137.035999046(27)	0.22σ

The normalization-free triangulation is the stronger structural result. Table 1 is included because it is the requested absolute two-protocol consequence of the canonical quadratic bridge; its status remains postdictive.

5.6 Existing Penning-trap holdout

The 2023 electron magnetic-moment measurement and Standard Model calculation infer [8]

$$a_{g-2} = 137.035999166(15).$$

Using the postdictively calibrated $\widehat{\delta}$,

$$\widehat{g}_{g-2} = \frac{\varepsilon_\emptyset - a_{g-2}}{\widehat{\delta}} = 1.04 \pm 0.19.$$

This is consistent with the closure-free class $g = 1$ at 0.22σ and with $g = 6/5$ at 0.83σ; current precision does not distinguish the classes. The datum was not used to construct the 6/5 ratio and is therefore a useful retrospective holdout, but not a blinded prospective test. The inferred a_{g-2} is not a direct apparatus-only readout: it is conditional on the adopted high-order QED coefficients and smaller hadronic and electroweak inputs. Past revisions of the eighth-order contribution and recent independent checks of the tenth-order coefficient illustrate that a theory-side update can shift the inferred α^{-1} without any change in Penning-trap topology [10, 11].

6 Registered prospective apparatus tests

The Rb–Cs pair calibrates the apparatus grammar postdictively. The numerical values in P1–P3 use one frozen amplitude only,

$$\boxed{B_{\mathrm{R}} = \varepsilon_\emptyset - a_{\mathrm{R}} = 7.94 \times 10^{-7},}$$

obtained from the Rb Raman readout alone. The Cs datum is not used to set this prospective Bragg-order line. The following four statements are frozen as prospective tests.

P1: same-apparatus Bragg-order sweep. For a Bragg readout of order n,

$$\boxed{a_n = \varepsilon_\emptyset - B_\mathrm{R}\left(1+\frac{1}{n}\right) = A - \frac{B}{n},}$$

with

$$A = \varepsilon_\emptyset - B_\mathrm{R} = a_\mathrm{R}, \qquad B = B_\mathrm{R}, \qquad A + B = \varepsilon_\emptyset.$$

This gives the frozen values in Table 2.

Table 2: Registered Bragg-order predictions using the Rb Raman readout as the one-time calibration.

Bragg order n	Predicted α_n^{-1}
1	137.0359984120000
3	137.0359989413333
4	137.0359990075000
5	137.0359990472000
6	137.0359990736667
10	137.0359991266000

A same-instrument sweep must be linear in $1/n$ and obey both endpoint constraints. A generic improvement of conventional systematics does not by itself imply this overdetermined line.

P2: two-species Raman/Bragg cross-over and the maximal $n = 1$ contrast. The factor follows readout topology, not atom species. An Rb Bragg implementation of order n must follow $1 + 1/n$, while a Cs Raman–Bloch implementation with the same closure-removing projection must follow $g = 1$. The sharpest registered contrast is first-order Bragg versus Raman: although both use a two-photon $2\hbar k$ momentum transfer, the topology grammar assigns

$$g_\mathrm{Bragg}(1) = 2, \qquad g_\mathrm{Raman} = 1,$$

and, under the frozen Rb calibration,

$$\boxed{a_{\mathrm{Bragg},1} = 137.0359984120.}$$

Failure of this topology-versus-kinematics contrast rejects the assignment rather than motivating a new rational factor.

P3: persistence at fixed fifth order. An improved $n = 5$ Cs measurement with substantially reduced wavefront and diffraction-phase uncertainties must remain near the frozen Rb-only prediction

$$\boxed{\varepsilon_\emptyset - \frac{6}{5}B_\mathrm{R} = 137.0359990472,}$$

rather than converge to the Raman class, provided the readout topology is unchanged.

P4: Penning-trap class. The electron $g - 2$ readout has no momentum ladder or Bragg multiport. The registered apparatus assignment is

$$\boxed{\chi_{g-2} = 0, \qquad g_{g-2} = 1.}$$

Conditional on the adopted and independently checked high-order QED coefficients and other Standard-Model inputs, future improved electron-moment determinations should approach the Raman-class readout and should not acquire an independently fitted rational apparatus factor. A later theory-side revision of those coefficients would shift the inferred a_{g-2} without changing apparatus topology and would therefore not, by itself, falsify the $\chi = 0$ assignment.

Printed withdrawal criterion

The apparatus-topology conjecture is retained only if one universal closure operator K_{cl}, inserted into the published Raman and Bragg transfer matrices, yields

$$\boxed{g_{\mathrm{Raman}} = 1,} \qquad \boxed{g_{\mathrm{Bragg}}(n) = 1 + \frac{1}{n}}$$

without apparatus-specific continuous constants and in addition to, rather than as a renaming of, the standard diffraction, wavefront, light-shift, and recoil corrections. If this pair is not obtained, or if P1–P4 fail, the apparatus-topology conjecture and its attractive Rb–Cs triangulation are to be withdrawn rather than repaired by a new channel-specific factor.

7 Charged-lepton mass-amplitude ratios

The charged-lepton relation introduced by Koide [12] is most naturally written in terms of mass amplitudes. Let $q_n = \sqrt{m_n}$ and let $\omega^3 = 1$. Consider

$$q_n = q_0 \left[1 + a\,\Re(\xi\omega^n)\right], \qquad n = 0, 1, 2.$$

Writing $x_n = \Re(\xi\omega^n)$ gives

$$\sum_n x_n = 0, \qquad \sum_n x_n^2 = \frac{3}{2}.$$

Hence

$$\sum_n q_n = 3q_0, \qquad \sum_n q_n^2 = q_0^2\left(3 + \frac{3}{2}a^2\right),$$

and

$$\boxed{K_{\mathrm{Koide}} = \frac{1}{3} + \frac{a^2}{6}.}$$

The C_3 identities alone do not fix the Koide value.

The quadratic comparator-norm axiom is now applied to the amplitude decomposition. The identity component $q_0(1,1,1)$ and the cyclic-deviation component $aq_0(x_0, x_1, x_2)$ are assigned equal squared norms. Their norms are $3q_0^2$ and $(3/2)a^2q_0^2$, so

$$a^2 = 2, \qquad a = \sqrt{2},$$

and therefore

$$\boxed{K_{\mathrm{Koide}} = \frac{2}{3}.}$$

The equivalent 45° geometry between the mass-amplitude vector and the democratic direction was noted by Foot [13]. For comparison, scalar-potential and Yukawaon nonet constructions also obtain modified Koide characters from flavor-nonet invariants, without the present closure-transfer interpretation [16]. The present contribution is not the geometric restatement itself, but the proposed common origin of the equal-norm condition in the background-independent counterpart axiom.

7.1 Transfer weight, phase embedding, and closure order

Graded closure covariance gives the information-transfer weight

$$s_{12} = \frac{2}{9}.$$

The coefficient has also appeared empirically in Z_3 charged-lepton mass parametrizations [14, 15].

Physical bridge hypothesis 7.1 (Oriented phase embedding)**.** *The transfer weight is embedded in the complex C_3 chart as*

$$\boxed{\vartheta_K = -\frac{2}{9}\ \textit{radian.}}$$

The radian embedding and its sign are not consequences of the transfer proposition.

Physical bridge hypothesis 7.2 (Closure-order assignment)**.** *The cyclic positions $n = 0, 1, 2$ are assigned to (τ, μ, e) and interpreted as the proposed $(1R, 2R, 3R)$ closure order.*

In the standard complex chart,

$$q_n = q_0 \left[1 + \sqrt{2} \cos\left(\vartheta_K + \frac{2\pi n}{3}\right)\right].$$

The factor $2\pi/3$ is a chart representation of C_3, not a primitive interior circumference.

For $\vartheta_K = -2/9$,

$$q_\tau : q_\mu : q_e = 2.37943817 : 0.58021192 : 0.04034991.$$

Table 3: Zeroth-order charged-lepton ratios. Observed ratios and propagated uncertainties use PDG 2026 charged-lepton masses [17]. The last column is shown symmetrically for all rows.

Ratio	Model	Observed	Difference	$\lvert\Delta\rvert/\sigma$
m_τ/m_μ	16.8180467	16.8176918	3.55×10^{-4}	0.42
m_τ/m_e	3477.47284	3477.36526	0.10758	0.61
m_μ/m_e	206.770316	206.768283	0.002033	452

The deep-generation ratio m_τ/m_μ is the successful zeroth-order postdiction and becomes a prospective test only through future, independently improved tau-mass measurements. The m_μ/m_e discrepancy is not hidden by a fitted tail term: it is a quantitative failure of the present zeroth-order electron postdiction/readout. A later full-$3R$ common-node derivation may change the electron component, but no correction direction or magnitude is claimed here.

8 Logical status and parameter accounting

Table 4: Logical status of the construction.

Item	Status	Content
Operational equivalence and $Z_0/Z_1/Z_2$	Definitions	Separate indistinguishability, structural alphabets, and realized measurements.
Quadratic comparator norm	Structural axiom	Equal-norm orthogonal counterparts imply $\Lambda = \sqrt{2}$.
$2^1, 3^2, 5^3$ alphabets	Structural grammar	Minimal non-aliasing cycles plus full-support closure.
Termination after $3T$	Structural hypothesis	Later distinctions occur inside a common translational domain.
Graded closure transfer	Internal proposition	Gives $2/9$ and $9/125$.
One-sheet weight $1/2$	Projector axiom	Gives the projected retention 0.036.

Item	Status	Content
$R_{\mathrm{id}} \mapsto \varepsilon_\emptyset$	Physical bridge	Identifies the weighted identity functional with the inverse-coupling asymptote 137.036.
$\varepsilon_E = 125$	Physical bridge	Identifies full 5^3 information density with the full-information endpoint.
Apparatus grammar g_A	Conjecture	Discrete closure-mode/rank map $1 + \chi_A/r_A$.
Rb and Cs assignments	Protocol hypotheses	$(\chi, r) = (0, 1)$ and $(1, 5)$, respectively.
Rb–Cs triangulation	Postdiction	Uses already published recoil data; no discovery significance is claimed.
$\delta_0 = 1125^{-2}$	Canonical bridge hypothesis	Gives the absolute double postdiction; not an internal theorem.
P1–P4	Registered prospective predictions	Frozen apparatus tests with a printed withdrawal criterion.
C_3 amplitude identities	Internal result	Give $K = 1/3 + a^2/6$.
$a = \sqrt{2}$	Application of the norm axiom	Equal identity and cyclic-deviation norms give Koide 2/3.
$\vartheta_K = -2/9$	Physical phase bridge	Radian embedding of the internally derived transfer weight.
(τ, μ, e) order	Closure-order hypothesis	Maps the cyclic positions to charged-lepton generations.
τ/μ ratio	Successful zeroth-order postdiction; prospective extension	Consistent with PDG 2026 at 0.42σ; future improved tau-mass data provide the prospective test.
μ/e ratio	Failed zeroth-order postdiction/readout	Differs by about 452σ; no fitted correction is introduced.

The apparatus sector contains one universal manifestation amplitude δ when calibrated postdictively, or none if the canonical δ_0 bridge is accepted. It contains no continuously adjustable factor for each apparatus. The lepton sector contains no fitted phase after the stated phase-embedding and closure-order hypotheses, but the electron readout remains incomplete.

9 Retrospective finite-grammar audit

To expose the remaining look-elsewhere dependence, we repeat a deliberately finite retrospective audit. This is not a discovery significance and does not exhaust all possible mathematical stories. Its purpose is to record which discrete choices have actually been searched.

The grammar fixes $p_1 = 2$ and allows

$$(p_2, p_3) \in \{(3,5), (3,7), (5,7)\}, \qquad w \in \left\{\frac{1}{2}, 1\right\}.$$

The comparator content is now varied explicitly:

$$N_{\mathrm{cmp}} \in \{N_2N_3,\ N_1N_2N_3,\ N_{\mathrm{id}}\}, \qquad N_{\mathrm{id}} := 1 + N_1 + N_2 + N_3.$$

The canonical readout exponent and sign are varied as

$$r \in \{1, 2, 3\}, \qquad s \in \{+1, -1\}.$$

For each candidate,

$$R = 1 + p_1 + p_2^2 + p_3^3 + w\frac{p_2^2}{p_3^3}, \qquad \delta_{\rm cand} = N_{\rm cmp}^{-r},$$

$$a_{\rm R,cand} = R - s\delta_{\rm cand}, \qquad a_{\rm C,cand} = R - \frac{6}{5}s\delta_{\rm cand}.$$

The lepton audit keeps the common quadratic norm $a = \sqrt{2}$, uses the direct oriented coordinate $\vartheta = -p_1/p_2^2$, and retains the fixed (τ, μ, e) cyclic assignment.

The grammar contains

$$N_{\rm grammar} = 3 \times 2 \times 3 \times 3 \times 2 = 108$$

candidates. With the explicitly retrospective tolerances

$$|a_{\rm R,cand} - a_{\rm R}| < 10^{-7},$$

$$|a_{\rm C,cand} - a_{\rm C}| < 10^{-7},$$

$$|(m_\tau/m_\mu)_{\rm cand} - (m_\tau/m_\mu)_{\rm obs}| < 10^{-3},$$

only one candidate survives:

$$\boxed{(p_2, p_3, w, N_{\rm cmp}, r, s) = \left(3, 5, \frac{1}{2}, N_2N_3, 2, +1\right).}$$

This updated audit includes the previously hidden choice between the surface–volume product, the full $1R \times 2R \times 3T$ product, and the additive identity count. It still fixes the unit normalization of the canonical bridge and the apparatus grammar $g_{\rm Rb} = 1$, $g_{\rm Cs} = 6/5$; it is therefore diagnostic bookkeeping, not an independent probability or a substitute for the transfer-matrix derivation.

10 Discussion

The fixed structural claim is the vacuum interval

$$\frac{1}{137.036} \le \alpha_\Phi \le \frac{1}{125}.$$

The apparatus conjecture does not renegotiate this interval. It asserts that internally measured precision readouts can carry discrete protocol-dependent manifestation factors. The current Rb–Cs tension is therefore used as a postdictive calibration point, not declared in advance to be a predicted anomaly.

The apparatus conjecture is stronger than a free statement that "measurements differ." It requires one small discrete grammar, forbids channel-by-channel rational fitting, predicts a same-device $1/n$ law with two endpoint constraints, and assigns the non-recoil Penning-trap channel to $\chi = 0$. Its weakest point is foundational: the pair $(g_{\rm Raman}, g_{\rm Bragg}) = (1, 1 + 1/n)$ has not yet been derived from the published pulse-sequence transfer matrices. The withdrawal criterion makes that dependence explicit.

The lepton result is likewise split. The C_3 algebra and equal-norm geometry yield the Koide character; the 2/9 transfer weight comes from closure covariance. The oriented radian embedding and generation assignment are physical hypotheses. Their strongest current consequence is the deep-generation τ/μ ratio. The electron-containing precision ratio is not described at the present order.

11 Conclusion

A background-independent finite vacuum grammar has been organized around operational equivalence, a stated quadratic comparator norm, three non-aliasing closure alphabets, and graded closure covariance. The construction yields

$$N_1 = 2, \qquad N_2 = 9, \qquad N_3 = 125,$$
$$\frac{N_1}{N_2} = \frac{2}{9}, \qquad \frac{N_2}{N_3} = \frac{9}{125},$$

and, through the vacuum readout bridge,

$$\varepsilon_{\emptyset} = 137.036, \qquad \varepsilon_E = 125.$$

The precision fine-structure sector is extended by a falsifiable apparatus-topology conjecture. The Raman–Bloch and fifth-order Bragg–Bloch protocols are assigned the discrete factors 1 and 6/5. The published Rb and Cs measurements then triangulate 137.036000006(150), and the canonical quadratic bridge gives a two-protocol postdiction agreeing at 0.35σ and 0.22σ. These are postdictions. The scientific risk is carried by four prospective tests and the requirement that one universal closure operator derive both apparatus factors without apparatus-specific continuous constants.

In the charged-lepton sector, the same quadratic comparator norm fixes the equal-norm amplitude coefficient $\sqrt{2}$, while closure covariance supplies 2/9. With an explicit phase embedding and closure-order assignment, the model gives a zeroth-order τ/μ postdiction consistent at 0.42σ. The μ/e ratio fails at the present order by about 452σ; no tail correction is added in this article.

The resulting construction is intentionally neither an assumption-free theorem nor an unconstrained numerical fit. It is a set of conditional deductions, one postdictively calibrated apparatus conjecture, four registered prospective tests, and an explicit rule for withdrawing the conjecture if its common transfer-matrix derivation or its experimental predictions fail.

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